# Research of Index Combination Weighting Model on Discipline Evaluation

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***Abstract*—The index combination weighting model based on the maximum entropy theory is proposed. This model takes the subjective factors as the goal planning body and transforms the subjective factors into the restrictions of target planning, which can achieve a good fusion of subjective and objective conditions. Taking the evaluation index system of three disciplines construction for instance, the effect of discipline construction is assessed by the model of combination weighting. The results show that this method is easy to implement on a computer, which has a high application value.**

***Keywords-evaluation index; index weighting modeling; discipline evaluation***

## I. INTRODUCTION

In western countries, some special assessment agencies have been developed, which evaluate the various disciplines regularly. For example, research doctorates are assessed by “American Research Council” and "the new U.S. Weekly” make evaluation for the disciplines and institutes regularly [1]. In our country, a lot of research work on the construction and assessment of disciplines has been carried out. The Ministry of Education has made examination and evaluation for national key disciplines for three times which has achieved significant results [2].

Indicator weighting is a focus of the discipline assessment models which can be divided into three methods, namely subjective weighting method, objective weighting method and combination weighting method [3]. Combination weighting method combines the subjective and objective factors, which can improve the authenticity and reliability of the assessment results. It has been a hot topic in the index weighting field and is considered as a more scientific and reasonable weighting method [4].

At present, combination weighting method mainly includes linear weighted combination weighting and multiplication synthetic weighting methods. The weight empowerment is really just a mechanical accumulation of the objective and subjective empowerment results. The obtaining of the objective and subjective weights are two separate processes and are not unified in the weighting process. It is not a fundamental solution to the shortcomings of the objective and subjective weighting methods and the weighting results have a common problem called “stiffness”.

In order to solve the defects of the existing combination weighting methods hereinbefore, based on the maximum entropy principle, this paper builds the model of index combination weighting combined with the characteristics of the index system of discipline construction. This model takes the subjective factors as the goal planning body and transforms the subjective factors into the restrictions of target planning, which can achieve a good fusion of subjective and objective factors.

## II. INDEX WEIGHT MODEL BASED ON MAXINUM ENTROPY

First, make the following assumptions: under certain constraints, two indicators are considered with the same importance degree if they can not be distinguished. It is the most subjective method of all the objective judgments. Taking a limit situation, various indicators in the evaluation index system are with the same importance degree, which means that the probability value is equal and this is when the "entropy" value is the maximum.

Based on the assumptions above, under certain objective and subjective constraints, it can ensure the relative fairness in the "importance" of the weights of various indicators to get the corresponding index weights by seeking the maximum "entropy" of object programming.

### *A.* The Establishment of Evaluation Matrix

Definition 1: Suppose $n$ assessment objects, denoted by $Q=\{Q_1,Q_2,\cdots,Q_n\}$ and $m$ evaluation objects, denoted by $P=\{P_1,P_2,\cdots P_m\}$ . $x_{ij}$ denotes the evaluation value of $Q_i$ to $P_j$ (the $j$-th index ), and $A$ is called the evaluation matrix of the assessed object to index set.

Definition 2: set up the "standard object" as $Q_{n-1}$, take the average index value of the $j$ $(j=1,2,\cdots,m)$ column of evaluation object as the corresponding value $x_{(n+1)j}$ of $j$ indicator of the standard objects.

$$x_{(n+1)j}=\frac{1}{n}\sum_{i=1}^{n}x_{ij} \tag{1}$$

Thus, the evaluation value of standard objects can be obtained. Add this value to matrix $A$, making it as the $n$+1 row and the extended evaluation matrix can be got as:

$$\overline{A}_{(n+1)m}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{(n+1)1} & x_{(n+1)2} & \cdots & x_{(n+1)m} \end{bmatrix} \tag{2}$$

*B. Valuation Matrix Quantitative Processing*

Since there is a big difference in dimensionless of various assessment indicators, they can not be weighted directly in the calculation process. To resolve this problem, data should be disposed so that they are with the same metrics, which is called quantitative processing. The relative processing method is adopted in this paper to dispose the evaluation matrix. Comparisons are made between the actual and average values of each index, getting the relative score value of each indicator, which achieves the goal of the same metrics.

When the quantitative process is made in $\overline{A}_{(n+1)m}$, matrix $R$ can be got as:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{3}$$

*C. The Establishment and Seek of Multi-objective Planning Modes*

*1)Goal Programming Equation*

Assuming that the weighting vectors of m assessment indicators are $W = (\omega_1, \omega_2, \cdots, \omega_m)^T$, the ultimate assessment value of *i*-th object is:

$$U_i = \sum_{j=1}^{m} \omega_j r_{ij} \tag{4}$$

where, $\omega_j$ is the proportion of the *j*-th indicator in indicator set. Using the maximum entropy idea to solve its weight, the goal programming equation can be established as:

$$\max H(\omega) = -\sum_{j=1}^{m} \omega_j \ln \omega_j \tag{5}$$

On the other hand, according to the definition of "standard object", the designed weight can be the most reasonable only when the weighted variance sum of the index values of assessed and standard objects is the least. As a result, another goal programming equation is created as follows:

$$\min \sum_{i=1}^{n} f_i(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_j (1 - r_{ij})^2 \tag{6}$$

*2) Establish Constraints*

Constraints are divided into objective constraints and subjective constraints based on its nature. The objective constraint conditions are as follows:

$$\sum_{j=1}^{m} \omega_j = 1, \quad \omega_j \geq 0 \tag{7}$$

It means that the accumulation of the assessment indicators is 1.

In the process of indicator weighting, assessment experts can draw the importance order of the assessment indicators based on their own knowledge and experience. The following subjective constraints can be got according to the advice of experts:

$$\omega_i > \omega_k, \text{ or } \omega_i \approx \omega_k, \text{ or } \omega_i = a\omega_k \tag{8}$$

where, $1 < i, \quad k \leq m$ and $i \neq m$, $a$ is a nonnegative value.

According to the above-mentioned goal equation as well as the subjective and objective constraints, the following mathematical model of multi-objective planning can be established:

$$\begin{cases} \max H(\omega) = -\sum_{j=1}^{m} \omega_j \ln \omega_j \\ \min \sum_{i=1}^{n} f_i(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_j (1 - r_{ij})^2 \\ \sum_{j=1}^{m} \omega_j = 1, \quad \omega_j \geq 0, \\ \omega_i > \omega_k, \quad 1 < i, \quad k \leq m, \quad i \neq m \end{cases} \tag{9}$$

To solve these two targets in the formula above, a mathematical model is constructed as follows:

$$\begin{cases} \min \delta \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_j (1 - r_{ij})^2 + (1 - \delta) \sum_{j=1}^{m} \omega_j \ln \omega_j \\ s.t. \quad \sum_{j=1}^{m} \omega_j = 1, \quad \omega_j \geq 0, \quad j = 1,2,\cdots m \end{cases} \tag{10}$$

where, $0 \leq \delta \leq 1$.

The target planning function will be sought and the following formula can be got:

$$\omega_j = \frac{\exp\left\{-\left[1 + \delta \sum_{i=1}^{n} (1 - r_{ij})^2 \Big/ (1 - \delta)\right]\right\}}{\sum_{j=1}^{m} \exp\left\{-\left[1 + \delta \sum_{i=1}^{n} (1 - r_{ij})^2 \Big/ (1 - \delta)\right]\right\}} \tag{11}$$

By changing the value of the $\delta$ parameter, different weighting results can be got, from which a group of weighting combination will be chosen and it is the one which is most consistent with the subjective constraints.

Finally, sort the evaluation objects according to the size of $U_i$ value in (4).

## III. SIMULATION ANALYSES

In this section, the performance of the disciplinary construction is assessed using the index weighting model proposed in this paper. First of all, the importance degree of each index can be got by dint of the experience and opinions of experts through the questionnaire, on the basis of access to relevant information. The primaries selection of indicators is finished, and then the evaluation indicator architecture of discipline building is constructed with the decomposition method, which is as shown in Fig. 1 [5].

In Fig. 1, the first-level index set of discipline construction is {Faculty building $B_1$, Personnel training $B_2$, Discipline research $B_3$, Discipline platform construction $B_4$, Academic exchanges $B_5$ }.

The second-level index set of faculty building $B_1$ = {Academician $C_1$, Changjiang scholar $C_2$, National talent project training targets $C_3$, Professor $C_4$, Provincial talent Project training objects $C_5$}, and the importance degrees of various indicators are 0.43, 0.21, 0.18, 0.1 and 0.08; The second-level index set of personnel training $B_2$ = {Doctor $C_6$, Graduate student $C_7$, Undergraduate $C_8$}, and the importance degrees of various indicators are 0.6, 0.3 and 0.1; The second-level index set of scientific research $B_3$ = {Monographs $C_9$, Patents $C_{10}$, EI $C_{11}$, Important journals $C_{12}$}, and the importance degrees of various indicators are 0.46, 0.32, 0.11 and 0.11; The second-level index set of disciplines platform construction $B_4$ = {National key laboratory $C_{13}$, National innovation team $C_{14}$, Doctoral section $C_{15}$, Provincial key laboratories $C_{16}$, Provincial innovation team $C_{17}$, Master section $C_{18}$}, and the importance degrees of various indicators are 0.26, 0.24, 0.16, 0.15, 0.12 and 0.07; The second-level index set of academic exchanges $B_5$ ={International academic conference $C_{19}$, National academic conference $C_{20}$, Deputy chairman of national institute $C_{21}$, Deputy chairman of provincial institute $C_{22}$}, and the importance degrees of various indicators are 0.37, 0.26, 0.23 and 0.14.

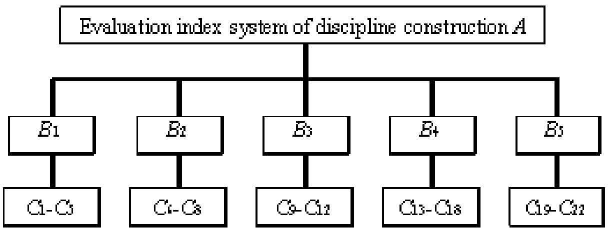


Figure 1. Evaluation index structure of discipline construction.

Five evaluated objects are selected to form a rated object set, denoted as $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$ and the corresponding index values of each evaluation object are shown in Table Ⅰ.

*A. Establish evaluatin matrix*

Constitute the evaluation matrix of evaluated object with various first-level indicators and the level of various first-level indicators can be got by multiplying the second-level indicators and their importance degrees. Well then, the building level of faculty can be expressed as:

$$B_1 = C_1 * 0.43 + C_2 * 0.21 + C_3 * 0.18 + C_4 * 0.10 + C_5 * 0.08$$

Sequence analogy, obtained:

$$B_2 = C_6 * 0.6 + C_7 * 0.3 + C_8 * 0.1$$

$$B_3 = C_9 * 0.46 + C_{10} * 0.32 + C_{11} * 0.11 + C_{12} * 0.11$$

$$B_4 = C_{13} * 0.26 + C_{14} * 0.24 + C_{15} * 0.16 + C_{16} * 0.15 + C_{17} * 0.12 + C_{18} * 0.07$$

$$B_5 = C_{19} * 0.37 + C_{20} * 0.25 + C_{21} * 0.24 + C_{22} * 0.14$$

In accordance with the preceding definition 1, the evaluation matrix constituted with 5 evaluated objects can be calculated as follows:

$$A = \begin{bmatrix} 3.03 & 67.7 & 9.08 & 3.27 & 4.05 \\ 3 & 56.8 & 8.17 & 2.28 & 2.03 \\ 2.79 & 59.5 & 11.4 & 1.33 & 2.29 \\ 2.38 & 45.7 & 8.65 & 2.26 & 1.03 \\ 2.28 & 44.4 & 5.74 & 1.22 & 1.28 \end{bmatrix}$$

*B. Establishes an Extended Evaluation Matrix*

Based on Definition 2 and Formula (1) and (2), the extended evaluation matrix $\overline{A}$ of matrix $A$ can be got as:

$$\overline{A} = \begin{bmatrix} 3.03 & 67.7 & 9.08 & 3.27 & 4.05 \\ 3 & 56.8 & 8.17 & 2.28 & 2.03 \\ 2.79 & 59.5 & 11.4 & 1.33 & 2.29 \\ 2.38 & 45.7 & 8.65 & 2.26 & 1.03 \\ 2.28 & 44.4 & 5.74 & 1.22 & 1.28 \\ 2.696 & 54.82 & 8.608 & 2.072 & 2.136 \end{bmatrix}$$

*C. Quantitative Processing*

Making a quantitative process with $\overline{A}$, matrix $R$ can be obtained:

$$R = \begin{bmatrix} 1.1239 & 1.235 & 1.0548 & 1.5782 & 1.8961 \\ 1.1128 & 1.0361 & 0.9491 & 1.1004 & 0.9504 \\ 1.0349 & 1.0854 & 1.3243 & 0.6419 & 1.0721 \\ 0.8828 & 0.8336 & 1.0049 & 1.0907 & 0.4822 \\ 0.8457 & 0.8099 & 0.6668 & 0.5888 & 0.5993 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Substituting the values in matrix R into the answering formula of $\omega_j$ and giving $\delta$ nine different values of 0.1 to 0.9, nine different weighting strategies can be got using the Matlab program.

Combining the subjective constraints $\omega_1 \approx \omega_2 > w_3 > \omega_4 > \omega_5$, select the optimal from these nine kinds of weighting strategies as the weight value. In this case it is the weight when $\delta = 0.4$:

$$(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)^{\mathrm{T}} = (0.2501, 0.2402, 0.2256, 0.1696, 0.1145)^{\mathrm{T}}$$

According to formula $U_i = \sum_{j=1}^{m} \omega_j r_{ij}$, the assessment values of the objects can be calculated as:

$$(Q_1, Q_2, Q_3, Q_4, Q_5)^{\mathrm{T}} = (1.3004, 1.0367, 1.0499, 0.8879, 0.7250)^{\mathrm{T}}$$

According to the calculations above, conclusions can be got: $Q_1 > Q_3 > Q_2 > Q_4 > Q_5$, which mean that the discipline construction level of $Q_1$ is the strongest and $Q_5$ is the worst. The instance analysis results are in line with the actual construction level of the selected disciplines, which denotes that the established model of weighting indicators is effective.

## IV. CONCLUSION

In this paper, the index weights are solved using the maximum entropy criterion and the first goal programming equation is set up. Then, based on the principle that the weighted variance between the evaluation and the standard objects is minimal, the second goal programming equation is set up. A new weighting model of indicator combination is constructed according to the thought of multi-target programming. It takes the subjective factors as the goal planning body and transforms the subjective factors into the restrictions of target planning, constituting a relatively close and perfect whole and solving the "stiffness" problem of objective weighting method caused by accumulating the objective and subjective weighting results mechanically. In addition, the index weighting model built in this paper has a highlight advantage that it pays attention to the whole in the process of weight design which takes the overall optimization as the starting point and the pursuit goal. Finally, the performance of subject building is assessed using this model. The results show that the method is precise, effective and easy to implement on a computer.


## ACKNOWLEDGMENT

This article is the results of the University Educational Quality Project of Liaoning Province (No. 2012106).


## REFERENCES


[1] Q. Song and G. Song, Analysis on discipline evaluation in colleges and universities, Journal of Shenyang Institute of Engineering, vol. 1, pp. 86-89, 2005.

[2] Y. Zhu and K. Yi, Thinking on further improvement of China first-class discipline ranking assessment, Sciences Research Management, vol. 27, no. 1, pp. 156-159, 2006.

[3] W. Li, Comprehensive index evaluation method and its application on water quality based on factor analysis, Journal of North University of China, vol. 32, pp. 207-211, 2011.

[4] Y. Zhang and Q. Bai, Environmental impact post evaluation for thermal power project based on combinational weight, East China Electric Power, vol. 38, no. 5, pp. 668-670, 2010.

[5] Y. Niu, W. Wu and S. Jia, On performance evaluation indicator system of discipline under new discipline director, Journal of North University of China, vol. 28, no. 1, pp. 52-58, 2012.


TABLE I. PERFORMANCE INDEX PARAMETERS OF DISCIPLINE CONSTRUCTION ASSESSMENT

| Third-level index | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | Third-level index | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Academician $C_1$ | 2 | 0 | 1 | 1 | 0 | Important journals $C_{12}$ | 21 | 26 | 51 | 34 | 17 |
| Changjiang scholar $C_2$ | 1 | 2 | 0 | 1 | 0 | National key laboratory $C_{13}$ | 4 | 3 | 1 | 2 | 1 |
| National talent project $C_3$ | 2 | 3 | 1 | 1 | 2 | National innovation team $C_{14}$ | 2 | 1 | 0 | 2 | 1 |
| Professor $C_4$ | 12 | 18 | 21 | 14 | 16 | Doctoral section $C_{15}$ | 3 | 1 | 2 | 3 | 2 |
| Provincial talent project $C_5$ | 5 | 3 | 1 | 2 | 4 | Provincial key laboratories $C_{16}$ | 5 | 4 | 2 | 3 | 2 |
| Doctor $C_6$ | 20 | 16 | 12 | 18 | 10 | Provincial innovation team $C_{17}$ | 2 | 3 | 2 | 1 | 1 |
| Graduate student $C_7$ | 85 | 64 | 73 | 54 | 46 | Master section $C_{18}$ | 4 | 2 | 3 | 3 | 2 |
| Undergraduate $C_8$ | 402 | 310 | 274 | 187 | 216 | International academic conference $C_{19}$ | 4 | 1 | 2 | 3 | 1 |
| Monographs $C_9$ | 5 | 3 | 6 | 5 | 2 | National academic conference $C_{20}$ | 34 | 26 | 28 | 31 | 29 |
| Patents $C_{10}$ | 4 | 3 | 5 | 3 | 2 | Deputy chairman of national institute $C_{21}$ | 4 | 2 | 3 | 0 | 1 |
| EI $C_{11}$ | 29 | 27 | 13 | 15 | 31 | Deputy chairman of provincial institute $C_{22}$ | 7 | 4 | 5 | 2 | 3 |