Gaussian Mixture Noise Improving Nonlinear Multiple Signal Detection*

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Abstract - Based on a nonlinear signal detector, stochastic resonance (SR) of noise-improved multiple signal detection is discussed for Gaussian mixture noise with different parameters. Signal transits from supra-threshold to sub-threshold when the threshold of the nonlinear signal detector is raised and noise transits from unimodal to bimodal when the parameter in Gaussian mixture noise increases, the occurrence of SR is studied in detail. The evolution of SR is a gradual process, which isn't affected completely by whether the signal is supra-threshold or sub-threshold, or by whether the noise is unimodal or bimodal. Based on computer simulation, for different fixed noise parameters and detector thresholds, the existence of SR shows a variety of circumstances and the detection efficacy can be improved rapidly when the sample number increases. These results show that the occurrence of SR depends strongly on the parameter in Gaussian mixture noise, the threshold of the nonlinear multiple signal detector and the sample number, and is their complex synergy.

Index Terms - multiple signal detection, probability of detection error, Gaussian mixture noise, stochastic resonance.

1. Introduction

Sometimes, noise can improve signal processing and information transmission in a nonlinear system. This phenomenon is called stochastic resonance (SR) [1-15], which is discovered firstly by R. Benzi et al. in 1981 [1]. Most studies of SR involve a sub-threshold signal (i.e., the signal cannot cross the system threshold without the assistance of noise) exciting a nonlinear system, which can produce a stronger beneficial response by adding suitable noise under certain conditions. In 2000, N. Stocks et al. explored firstly supra-threshold stochastic resonance (SSR) of noise-assisted supra-threshold signal (i.e., a continuously valued random signal) processing based on a summing network of threshold devices [16]. Without noise, the supra-threshold signal usually elicits the same response from each of the threshold devices. Noise often deteriorates supra-threshold signal transmission in a single system, however, when different noises are independently added on these devices of the summing network, they will produce distinct responses, when all these responses are summed, the overall response can be more efficient than the response in every threshold device, which shows that noise is beneficial to supra-threshold signal processing [16-29]. SSR extended the mechanism of SR. The study of SR and SSR has been an active field in the study of the effects played by noise and nonlinearity. People with great interest have studied the existence, theories, simulations, experiments and applications of SR and SSR in different fields and from different aspects. SR and SSR are usually characterized by signal-to-noise ratio (SNR), mutual information (MI), cross-correlation coefficient (CCC), probability of detection (P_d), and so on [1-29]. The study of SR and SSR in binary signal detection has also received many attentions [6,12,13,25,28-48]. In [11], the problem of non-Gaussian noise improving binary signal detection has been preliminarily explored, for unimodal generalized Gaussian noise, only when the input signal is sub-threshold, the noise can enhance the signal detection and SR exists, however, for bimodal Gaussian mixture noise, not only when the input signal is sub-threshold, SR often exists, but also when the input signal is supra-threshold, SR sometimes exists, too. In [49], we discussed SR in nonlinear multiple signal detection for four representative unimodal noises (Uniform noise, Gaussian noise, Laplace noise and Cauchy noise) and obtained the similar result for unimodal generalized Gaussian noise in binary signal detection, namely, only when the signal is sub-threshold, noise can enhance the signal detection and SR exists. Here, we further discuss in detail SR in the nonlinear multiple signal detection for Gaussian mixture noise. The existence and the efficiency of SR rely intensely on noise parameter, detector threshold and the sample number, and is the result of their common action in nonlinear multiple signal detection.

2. Nonlinear Multiple Signal Detection

As in [49], s is a random ternary signal and takes the constant value s_{-1} (hypothesis H_{-1}), s_0 (hypothesis H_0) or s_1 (hypothesis H_1) with the equal prior probabilities $P_j = 1/3\,(j = -1, 0, 1)$. An independent noise θ is added to the original signal s. z is the output of a three-state nonli-nearity

$$z = \begin{cases} -1, & s+\theta < -u \\ 0, & -u < s+\theta < u, \\ 1, & s+\theta > u \end{cases} \qquad (1)$$

where u is a adjustable threshold. The ternary data set $\mathbf{Z} = (z_1, z_2, \cdots, z_N)$ is the new sample observation. Then the nonlinear sum statistic $z_s = \sum_{i=1}^{N} z_i$ assumes integer values between $-N$ and N. ($z_s \in \{-N, -N+1, \cdots, 0, \cdots, N-1, N\}$).

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The maximum a posterior probability (MAP) detector by the sum statistic z_s can be obtained based on the likelihood ratios

$$\frac{P_0 \cdot \Pr(z_s = n \mid H_0)}{P_{-1} \cdot \Pr(z_s = n \mid H_{-1})} \underset{H_{-1},H_1}{\overset{H_0,H_1}{\gtrless}} 1, \tag{2}$$

$$\frac{P_1 \cdot \Pr(z_s = n \mid H_1)}{P_{-1} \cdot \Pr(z_s = n \mid H_{-1})} \underset{H_{-1},H_0}{\overset{H_0,H_1}{\gtrless}} 1, \tag{3}$$

$$\frac{P_1 \cdot \Pr(z_s = n \mid H_1)}{P_0 \cdot \Pr(z_s = n \mid H_0)} \underset{H_{-1},H_0}{\overset{H_{-1},H_1}{\gtrless}} 1. \tag{4}$$

The minimal P_{er} of the MAP detector is

$$P_{er} = \sum_{n=-N}^{N} \big[P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) + P_0 \cdot \Pr(z_s = n \mid H_0) + P_1 \cdot \Pr(z_s = n \mid H_1)\big] - \max\{P_{-1} \cdot \Pr(z_s = n \mid H_{-1}), P_0 \cdot \Pr(z_s = n \mid H_0), P_1 \cdot \Pr(z_s = n \mid H_1)\} \tag{5}$$

$$= \sum_{n=-N}^{N} \frac{3}{4}\big[P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) + P_1 \cdot \Pr(z_s = n \mid H_1)\big] + \frac{1}{2}P_0 \cdot \Pr(z_s = n \mid H_0) - \frac{1}{4}\left|P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) - P_1 \cdot \Pr(z_s = n \mid H_1)\right| - \left|\frac{1}{4}\big[P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) + P_1 \cdot \Pr(z_s = n \mid H_1)\big] + \frac{1}{4}\left|P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) - P_1 \cdot \Pr(z_s = n \mid H_1)\right| - \frac{1}{2}P_0 \cdot \Pr(z_s = n \mid H_0)\right| \tag{6}$$

$$= \frac{1}{2} + \frac{1}{4}\sum_{n=-N}^{N} \big[P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) + P_1 \cdot \Pr(z_s = n \mid H_1)\big] - \frac{1}{4}\sum_{n=-N}^{N} \left|P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) - P_1 \cdot \Pr(z_s = n \mid H_1)\right| - \sum_{n=-N}^{N} \left|\frac{1}{4}\big[P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) + P_1 \cdot \Pr(z_s = n \mid H_1)\big] + \frac{1}{4}\left|P_{-1} \cdot \Pr(z_s = n \mid H_{-1}) - P_1 \cdot \Pr(z_s = n \mid H_1)\right| - \frac{1}{2}P_0 \cdot \Pr(z_s = n \mid H_0)\right|. \tag{7}$$

where (6) can be obtained via

$$\begin{aligned}\max\{a,b,c\} &= \max\{\max\{a,b\},c\} \\ &= \max\{\tfrac{1}{2}(a+b) + \tfrac{1}{2}|a-b|, c\} \\ &= \frac{1}{4}(a+b) + \frac{1}{4}|a-b| + \frac{c}{2} \\ &\quad + \left|\frac{1}{4}(a+b) + \frac{1}{4}|a-b| - \frac{c}{2}\right|.\end{aligned} \tag{8}$$

Given the noise θ with the probability density function (PDF) $f(x)$ and the cumulative distribution function (CDF) $F(x)$. Then the conditional probabilities are

$$\Pr(z_i = -1 \mid H_k) = \Pr(\theta + k < -u) = \Pr(\theta < -u - k) = 1 - F(u+k) = \begin{cases} 1 - F(u-1), & k = -1 \\ 1 - F(u), & k = 0 \\ 1 - F(u+1), & k = 1 \end{cases}, \tag{9}$$

$$\Pr(z_i = 1 \mid H_k) = \Pr(\theta + k > u) = \Pr(\theta > u - k) = 1 - F(u-k) = \begin{cases} 1 - F(u+1), & k = -1 \\ 1 - F(u), & k = 0 \\ 1 - F(u-1), & k = 1 \end{cases}, \tag{10}$$

$$\Pr(z_i = 0 \mid H_k) = F(u+k) + F(u-k) - 1 = \begin{cases} F(u-1) + F(u+1) - 1, & k = -1 \\ 2F(u) - 1, & k = 0 \\ F(u+1) + F(u-1) - 1, & k = 1 \end{cases}. \tag{11}$$

For one sample ($N = 1$) special case, P_{er} in (7) is simplified as

$$P_{er} = \frac{2}{3} - \frac{1}{6}F(u+1) - \frac{1}{3}F(u) + \frac{1}{2}F(u-1) - \frac{1}{6}\left|F(u-1) + F(u+1) - 2F(u)\right|. \tag{12}$$

3. Gaussian Mixture Noise Improving Multiple Signal Detection

Gaussian mixture noise PDF with zero mean and two parameters (m, σ) is [12]

$$f(x)=\frac{1}{2}\frac{1}{\sqrt{2\pi(\sigma^2-m^2)}}\left[\exp\left(-\frac{(x-m)^2}{2(\sigma^2-m^2)}\right)+\exp\left(-\frac{(x+m)^2}{2(\sigma^2-m^2)}\right)\right],\sigma>m. \tag{13}$$

And CDF is

$$\begin{aligned}F(x)&=\int_{-\infty}^{x}f(x)dx\\&=\int_{-\infty}^{x}\frac{1}{2}\frac{1}{\sqrt{2\pi(\sigma^2-m^2)}}\left[\exp\left(-\frac{(x-m)^2}{2(\sigma^2-m^2)}\right)+\exp\left(-\frac{(x+m)^2}{2(\sigma^2-m^2)}\right)\right]dx\\&=\frac{1}{2}+\frac{1}{4}erf\left(\frac{x-m}{\sqrt{2(\sigma^2-m^2)}}\right)+\frac{1}{4}erf\left(\frac{x+m}{\sqrt{2(\sigma^2-m^2)}}\right),\\&\qquad\sigma>m,-\infty<x<\infty.\end{aligned} \tag{14}$$

When $m=0$, the Gaussian mixture noise is just Gaussian noise. Gaussian mixture noise models are widely used in ocean acoustics and sonar applications, and often used in the study of SR and SSR.

By inserting (14) into (12) , we can obtain

$$\begin{aligned}P_{er}=&\frac{2}{3}-\frac{1}{24}\left[erf\left(\frac{u+1-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u+1+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\\&-\frac{1}{12}\left[erf\left(\frac{u-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\\&+\frac{1}{18}\left[erf\left(\frac{u-1-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u-1+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\\&-\frac{1}{24}\left|\left[erf\left(\frac{u-1-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u-1+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\right.\\&+\left[erf\left(\frac{u+1-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u+1+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\\&\left.-2\left[erf\left(\frac{u-m}{\sqrt{2(\sigma^2-m^2)}}\right)+erf\left(\frac{u+m}{\sqrt{2(\sigma^2-m^2)}}\right)\right]\right|.\end{aligned} \tag{15}$$

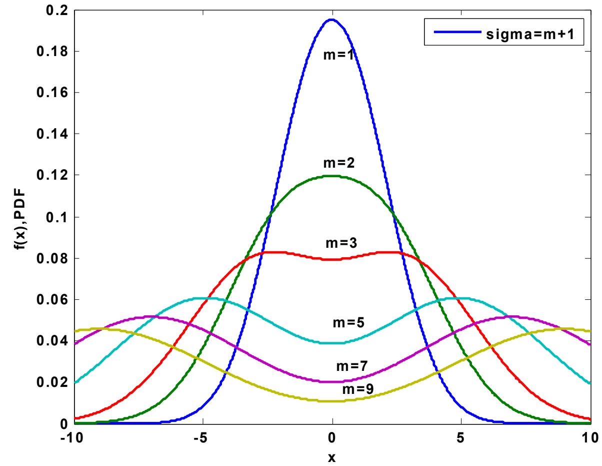

Fig.1 Gaussian mixture noise PDFs with different parameters.

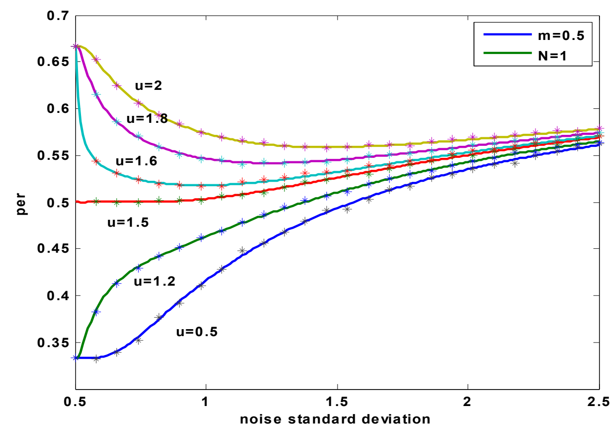

Fig.2 The probability of error P_{er} , as a function of the Gaussian mixture noise intensity σ, for $m=0.5$, $N=1$ and different threshold levels.

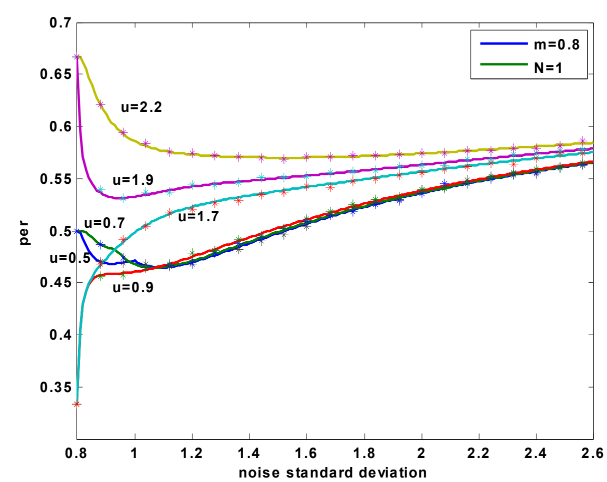

Fig.3 The probability of error P_{er} , as a function of the Gaussian mixture noise intensity σ, for $m=0.8$, $N=1$ and different threshold levels.

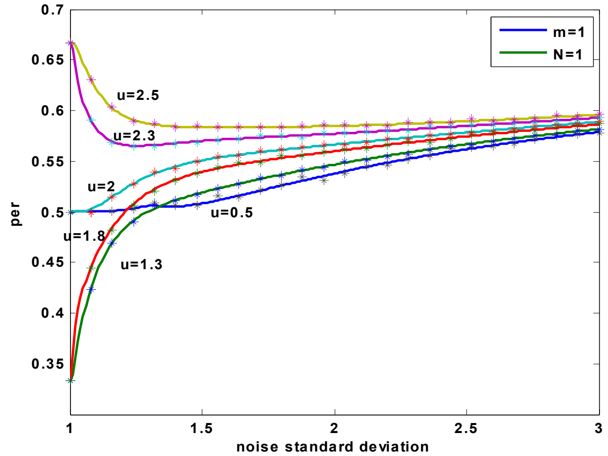

Fig.4 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=1$, $N=1$ and different threshold levels.

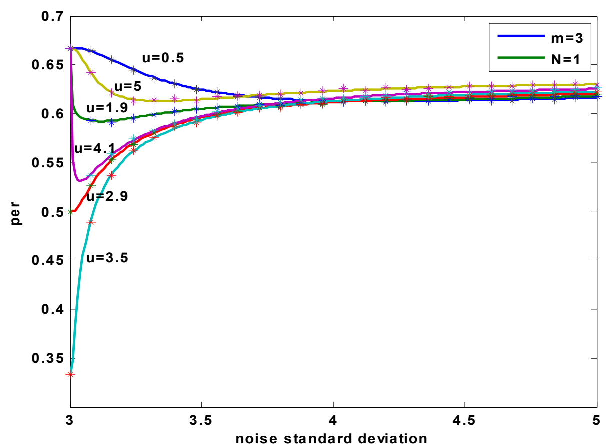

Fig.5 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=3, N=1$ and different threshold levels.

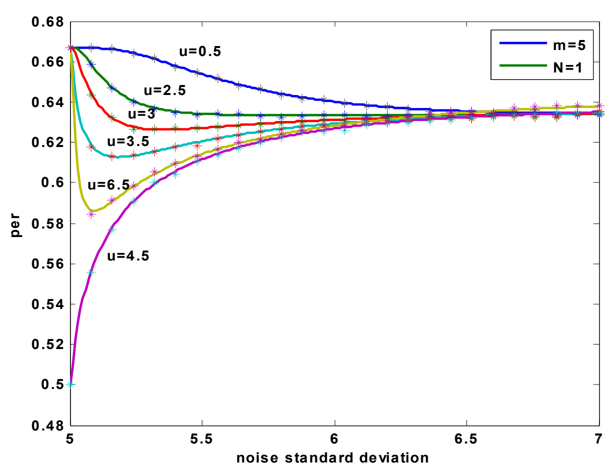

Fig.6 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=5, N=1$ and different threshold levels.

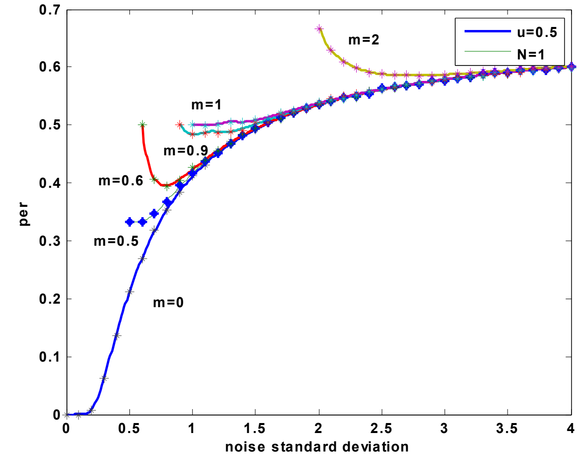

Fig.7 The probability of error P_{er}, as a function of the intensity σ for Gaussian mixture noise with different noise parameters and $u=0.5, N=1$.

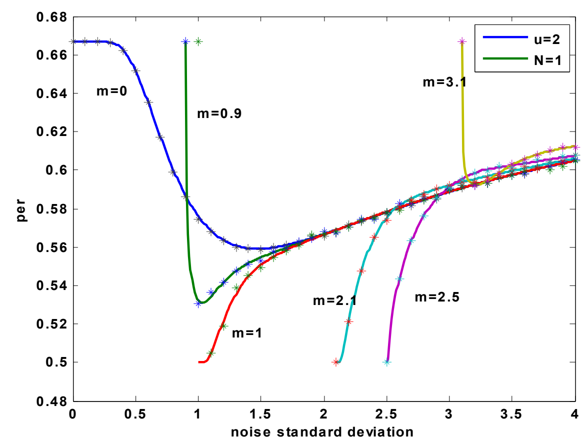

Fig.8 The probability of error P_{er}, as a function of the intensity σ for Gaussian mixture noise with different noise parameters and $u=2, N=1$.

Fig.1 gives Gaussian mixture noise PDFs for different parameters $(m=1,2,3,5,7,9)$. When the parameter m raises, the noise PDFs transit gradually from unimodal to bimodal.

For Gaussian mixture noise with different parameters and the nonlinear detector with different threshold levels, Figs.2-8 show the theoretical curves and the Monte Carlo simulation (data points) of the probability of detection error P_{er} as a function of noise intensity σ. Figs.2-8 confirm the agreement of the nonlinear multiple signal detection both in theory and the computer simulation. When the detector threshold level increases, the signal always transits from supra-threshold ($u<1$) to sub-threshold($u \geq 1$), when $m=0.5,0.8,1$, the Gaussian mixture noise PDFs are unimodal, when $m=3,5$, the Gaussian mixture noise PDFs are bimodal, Fig.2 and Fig.4 show the evolution of SR of noise-improved signal detection grows out of nothing, however, Fig.3, Fig.5 and Fig.6 also show the evolution of SR: from appearance to disa-ppearance and return to appearance. For a definite noise parameter m, Figs.2-6 also show that the occurrence of SR is a gradual process, which isn't affected completely by whether the signal is supra-threshold or sub-threshold, and these results are different from the previous ones about SR in multiple signal detection for four representative noises [49], where SR always exists when the signal is sub-threshold and SR doesn't exist when the input signal is supra-threshold.

As the parameter m increases from zero to infinity, the Gaussian mixture noise PDF changes from unimodal to bimodal, when the signal is supra-threshold $(u=0.5)$, Fig.7 shows SR has an evolution: from disappearance to appearance and then return to disappearance but then appearance. However, when the signal is sub-threshold $(u=2)$, Fig.8 shows another SR evolution: from appearance to disappearance, and then return to appearance. Fig.7 and Fig.8 also show the evolution of SR is a gradual process, which isn't affected completely by whether the noise is unimodal or bimodal. Figs.2-8 all show that P_{er} tends to consistency when the noise intensity becomes stronger. This is because stronger noises usually induce linearization [16].

For Gaussian mixture noise with different parameters and the nonlinear detector with different threshold levels, Figs.9-14 give the Monte Carlo computer simulation of P_{er} as a function of the noise intensity for different sample numbers. For the Gaussian mixture noise with parameter $m=0.5$, Fig.9 and Fig.10 show that SR don't occur no matter the input signal is supra-threshold $(u=0.5<1)$ or sub-threshold $(u=1.2>1)$ and whatever the sample number N is. For the Gaussian mixture noise with $m=0.8$ and $m=3$, Figs.11-14 show that SR always occurs and the minimum of P_{er} even can approach zero rapidly as N increases, regardless of the existence of SR when $N=1$. In Fig.11and Fig.13, SR doesn't exist when $N=1$, but as N increases, SR appears. The phenomenon is a bit like SSR [16]. Although the input is a discrete random signal and differs from the one in [16], where the input is a continuously valued randon signal, the mechanism of SR may be in accordance with the mechanism of SSR proposed by N.Stocks et al. . Figs.9-14 also confirm that no matter whether SR appears or not, the detection performance becomes better and better as the sample number N increases.

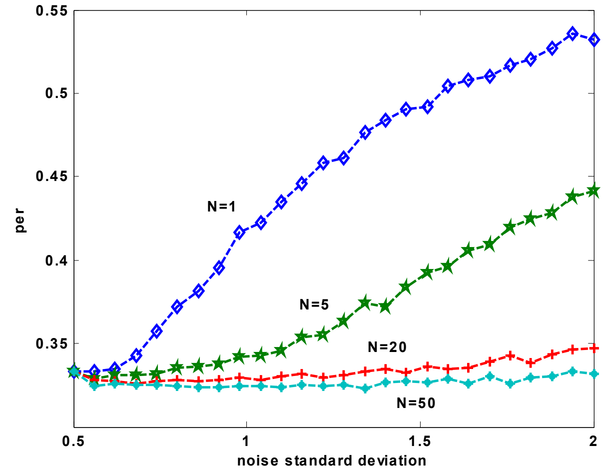

Fig.9 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=0.5, u=0.5$ and different sample numbers.

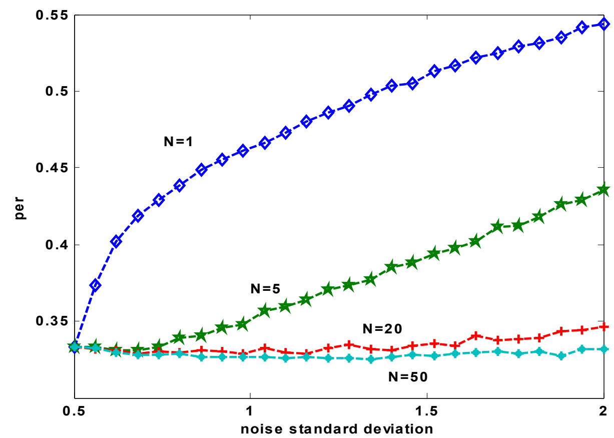

Fig.10 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=0.5, u=1.2$ and different sample numbers.

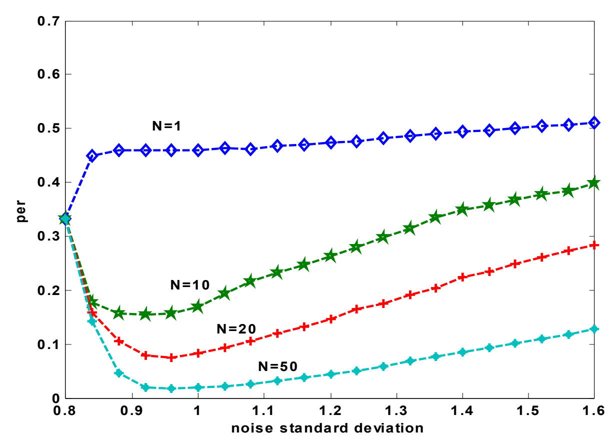

Fig.11 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m=0.8, u=0.9$ and different sample numbers.

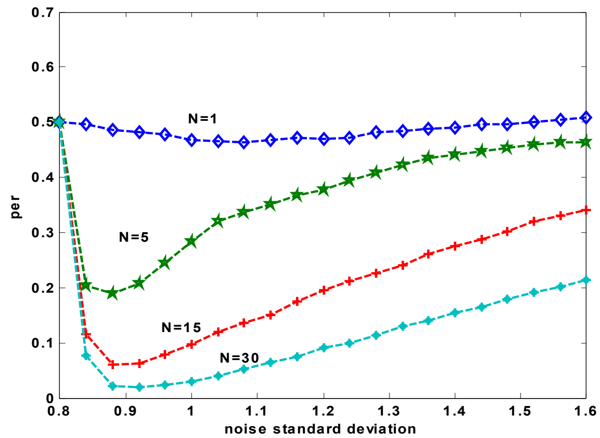

Fig.12 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m = 0.8, u = 0.7$ and different sample numbers.

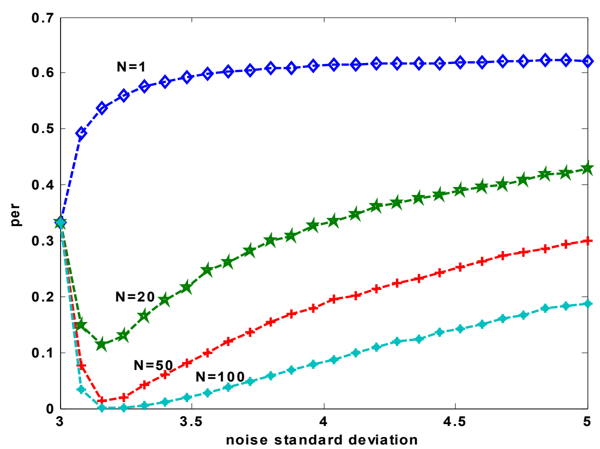

Fig.13 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m = 3, u = 3.5$ and different sample numbers.

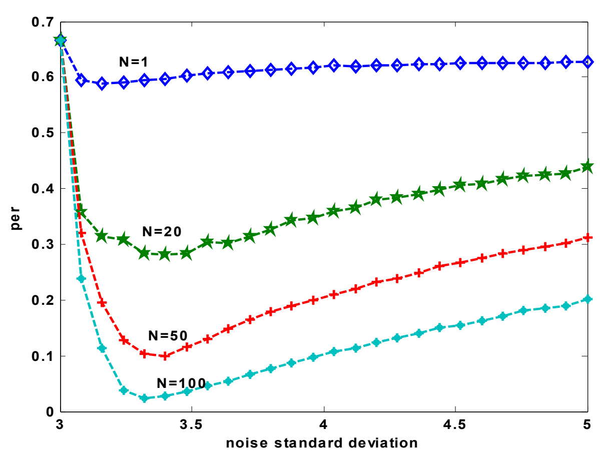

Fig.14 The probability of error P_{er}, as a function of the Gaussian mixture noise intensity σ, for $m = 3, u = 4.5$ and different sample numbers.

4. Conclusions

In this paper, we have discussed in detail SR pheno-menon of noise-improved nonlinear multiple signal detection under the background of Gaussian mixture noise. Based on detection theory and computer simulation, we have discussed that the parameter in Gaussian mixture noise PDF, the threshold level of the nonlinear multiple signal detector and the sample number all have effects on the existence of SR and the detection performance. These results indicate that SR relies intensely on noise, detector and the sample number and is the result of their complex synergy in nonlinear multiple signal detection.

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