If root u is in Q_{n-1}^{1}, for $i=1$ to $n-1$, we let $W_i=\{1u_{n-1}u_{n-2}...\overline{u_i}x_{i-1}x_{i-2}...x_0\in V(Q_{n-1}^{1})\}$. Otherwise, we let $W_i=\{0u_{n-1}u_{n-2}...\overline{u_i}x_{i-1}x_{i-2}...x_0\in V(M_{n-1}^{0})\}$. It is easy to construct ISTs $T_{A,1}$, $T_{A,2}$, and $T_{A,3}$ rooted at 1 in M_3^0. By Algorithm XIST, we can obtain $T_{B,1}$, $T_{B,2}$, and $T_{B,3}$ rooted at node 9 in Q_3^1. Fig. 3(a) shows how to construct four ISTs T_1, T_2, T_3, and T_4 rooted at node 1 or 9 in HMH_4 based on three ISTs $T_{A,1}$, $T_{A,2}$, and $T_{A,3}$ rooted at node 1 in M_3^0, three ISTs $T_{B,1}$, $T_{B,2}$, and $T_{B,3}$ rooted at node 9 in Q_3^1, and an ENSP {{0},{2,3},{4,5,6,7},{1}} on M_3^0. Fig. 3(b) illustrates the forward direction example which shows the four ISTs rooted at 1. Similarly, the four ISTs rooted at 9 with the backword direction are shown in Fig. 3(c).

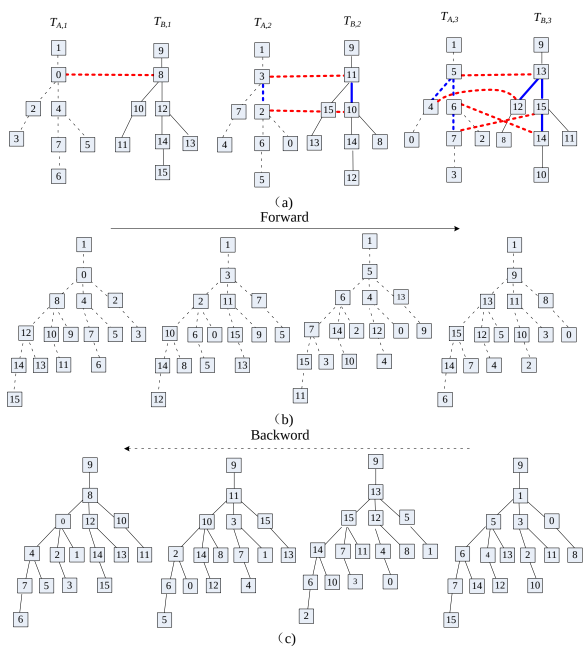

Fig. 3 Construction procedures of ISTs in HMH_4.

Based Definition 5 and the above discussion, we have the following lemma.

Lemma 3. $HMH_n\in\Gamma_n$.

Based on Lemma 2, Lemma 3, and the above discussion, we have the following theorem.

Theorem 2. $\Gamma_n\supseteq\{HMH_n,Q_n,LTQ_n,TQ_n,CQ_n,MQ_n,\quad,HCH_n\}$.

V . Conclusions

In this paper, we have proposed a more general apporoch to construct n ISTs in a family of of hypercube variants——conditional BC networks, which not only include the existing hypercube variants such as Q_n, LTQ_n, TQ_n, CQ_n, MQ_n, HCH_n, but also include some other ones such as HMH_n.

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