Theorem 3: The linear span of sequence $s_{u,v,w}$ in $S^{(r)}$ is given by

$$LS(s_{u,v,w}) = n \cdot l^{(n-2)/6} / 2\,,\ l = 2,3,4,5,6,7\,.$$

Proof: Since $n = 2m$, we have $LS(s_{u,v,w}) = m \cdot l^{(m-1)/3}$ $= n \cdot l^{(n-2)/6} / 2$, where $l = 2,3,4,5,6,7$. Thus we are done.

Combining Lemma 2, 3, 4 and Theorem 3, the linear span of sequences in $S^{(r)}$ has the following distribution.

$$LS(s_{u,v,w}) = \begin{cases} n \cdot 2^{(n-2)/6} / 2, & 1 \text{ time} \\ n \cdot 3^{(n-2)/6} / 2, & 2^{n/2} - 1 \text{ times} \\ n \cdot 4^{(n-2)/6} / 2, & 2(2^n - 1) \text{ times} \\ n \cdot 5^{(n-2)/6} / 2, & 2(2^{n/2} - 1)(2^n - 1) \text{ times} \\ n \cdot 6^{(n-2)/6} / 2, & (2^n - 1)^2 \text{ times} \\ n \cdot 7^{(n-2)/6} / 2. & (2^n - 1)^2 (2^{n/2} - 1) \text{ times} \end{cases}$$

IV. Conclusion

In this paper, a new family of binary sequences S with large size is proposed. This sequence family has maximum nontrivial correlation value $5 \cdot 2^{n/2} - 1$, maximum linear span $n \cdot 7^{(n-2)/6} / 2$ and family size $2^{5n/2}$. Compared with several previously known large families of binary sequences, this new sequence family not only has large size and low correlation, but also large linear span. And it can be a candidate in some applications where the family size and linear span are more important than the correlation value. In addition, it is a very interest in calculating the linear span of the sequence for a general value of parameter *r*, and it is an open problem to determine the distribution of correlation values of this sequence family.

References

[1] M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt. Spread Spectrum Communications Handbook, McGraw Hill, 2001.

[2] Kasami T. Weight distribution of Bose-Chaudhuri-Hocquenghem codes , Combinatorial Mathematics and Its Applications, Chapel Hill, NC: Univ. North Carolina Press, 1969.

[3] R. Gold. Maximal recursive sequences with 3-valued recursive crosscorrelation functions, IEEE Trans. Inf. Theory, vol.14, no.1, pp.154-156, 1968.

[4] J. D. Olsen, R. A. Scholt, and L. R. Welch, Bent function sequences, IEEE Trans. Inf. Theory, vol.IT-28, no.6, pp.858-864, 1982.

[5] A. Chang, et al. "On a conjectured ideal autocorrelation sequence and a related triple-error correcting cyclic code", IEEE Trans. Inf. Theory, vol.46, no.2, pp.680-687, Mar. 2000.

[6] O. S.Rothaus. "Modified Gold codes", IEEE Trans. Inf. Theory, vol.39, no.2, pp.654-656, Mar. 1993.

[7] N. Y. Yu and G. Gong. A new binary sequence family with low correlation and large size,

[8] Z. C. Zhou and X. H. Tang. New families of binary sequences with low correlation and large size, IEICE Trans. Fundamentals vol. E92-A, no.1, pp.291-297, 2009.

[9] Fanxin Zeng. "Two classes of large families of sequences with low correlation", IEEE Proceedings of IWSDA'97, pp.56-60.

[10] J. S. No and P. V. Kumar. A new family of binary pseudorandom sequences having optimal periodic correlation properties and large linear span, IEEE Trans. Inf. Theory, vol.35, no.2, pp.371-379, 1989.

[11] A. Klapper, d-form sequences: Familes of sequences with low correlation values and large linear spans, IEEE Trans. Inf. Theory, vol. 41, no.2, pp.654-656, 1995.

[12] X. Y. Zeng, L. HU and W. F. Jiang. A family of binary sequences with 4-valued optimal out-of-phase correlation and large linear span, IEICE Trans. Fundamentals, vol. E89-A, no.7, 2006.

[13] Niho Y. Multivalued Cross-Correlation Functions between Two Maximal Linear Recursive Sequences, Ph. D. dissertation, Univ. Southern Calif., Los Angeles, 1972.

[14] T. Helleseth. Some results about the cross-correlation fuction between two maximal linear sequences. Discrete Mathmatics, 16(3): 209-232, 1976.

[15] R. Lidl, H. Niederreiter, Finite Fields in Encyclopedia of Mathematics and Its Applications, vol. 20, Addison-Wesley, Reading, MA, 1983.

[16] E L. Key, "An analysis of the structure and complexity of non-linear binary sequence generators", IEEE Trans. Theory, vol. IT-22, pp. 732-736, 1976.