$$
\mathbf{H_{bb}} = \begin{pmatrix} H_{5,5} & \vdots & H_{5,3} & H_{5,4} & H_{5,n+6} & H_{5,n+7} \\ \cdots & \vdots & \cdots & \cdots & \cdots & \cdots \\ H_{3,5} & \vdots & H_{3,3} & H_{3,4} & H_{3,n+6} & H_{3,n+7} \\ H_{4,5} & \vdots & H_{4,3} & H_{4,4} & H_{4,n+6} & H_{4,n+7} \\ H_{n+6,5} & \vdots & H_{n+6,3} & H_{n+6,4} & H_{n+6,n+6} & H_{n+6,n+7} \\ H_{n+7,5} & \vdots & H_{n+7,3} & H_{n+7,4} & H_{n+7,n+6} & H_{n+7,n+7} \end{pmatrix} \quad (49)
$$

IV. Numerical examples

Let us consider a single-walled nanotube having the same geometrical and physical properties used in Kiani [5]: R=3 nm in mean radius, t=0.34 nm wall thickness, E=1TPa, Young's modulus and mass density $\rho=2.5\times10^3 kg/m^3$. Moreover, the non dimensional coefficient $\lambda=L/r_b$, with $r_b=(I/A)^{1/2}$, is assumed.

Table 1: Simply supported-simply supported SWCNT

λ	η=0.1			η=0.3		
	NEBT	[15]	DQM	NEBT	[15]	DQM
10	3.0006	2.9972	2.9185	2.6208	2.6177	2.6177
	5.3234	5.3202	5.3202	3.9601	3.9580	3.9580
	6.8659	6.8587	6.8583	4.6463	4.6426	4.6424
	7.8384	7.8248	7.8434	5.0270	5.0210	5.0290
	8.4568	8.4356	8.6382	5.2530	5.2446	5.3129
30	3.0638	3.0602	3.0602	2.6413	2.6727	2.6727
	5.7237	5.7199	5.7200	4.0803	4.2554	4.2554
	7.8633	7.8530	7.8525	4.9058	5.3157	5.3153
	9.5478	9.5233	9.5570	5.4266	6.1110	6.1305
	10.8820	10.8346	10.3530	5.7775	6.7361	6.9669

In Table 1, the first five non dimensional free frequencies $\Lambda_i=(\omega_i^2\times\rho AL^4/EI)^{1/4}$ of simply supported-simply supported SWCNT are reported, for varying values of the non dimensional coefficient λ= (10,30) and of the parameter η = (0.1,0.3) while the elastic medium parameters are assumed to be zero. The results have been calculated using a general code, developed in *Mathematica* [14]. A numerical comparison, illustrated in Table 1, is furnished between the results given by the proposed method (DQM) and the results given by Kiani [5,15] by means NEBT method. As one can see there is a good agreement between the two numerical approaches.

In Figure 1, the free frequencies of clamped-clamped SWCNT, having the same geometric and mechanical properties of the above example, are reported. Fixed λ=10 and η = 0.3, six curves have been plotted for different values of Winkler-type parameter K_{w1}= [0, 1, 10, 50, 100] of equation (11). On the axis x the values of K_{p1}, (see equation 11) are reported, while on the axis y the Λ_1 values are listed. As can be seen, the fundamental frequencies increase the Pasternak parameter and Winkler coefficients.

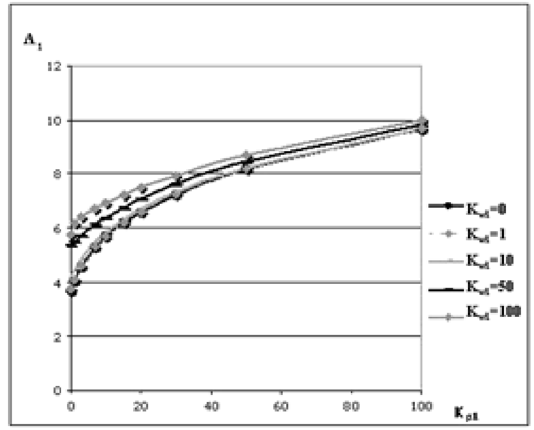

Fig.1: Clamped-Clamped SWCNT

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