Robust Mean Square Exponential Stability of Stochastic Interval Cellular Neural Networks with Time-delays

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Abstract - The problem of robust mean-square exponential stability for a class of stochastic interval cellular neural networks with time-delays is investigated. Firstly, a kind of equivalent description of this stochastic interval delayed cellular neural networks is presented. Then by using the $It\hat{o}$ formula, Razumikhin theorems, Lyapunov function and norm inequalities, several simple sufficient conditions are obtained which guarantee the robust mean-square exponential stability of the stochastic interval cellular neural networks. and some recent results reported in the literatures are generalized.

Index Terms - Robust Mean Square, Neural Networks, Time-delays.

I. Introduction

In recent years, neural networks have been extensively investigated, and successfully applied in many areas such as combinatorial optimization, signal processing, pattern recognition and many other fields. However, all successful applications are greatly dependent on the dynamic behaviors of neural networks. As is well-known now, stability is one of the main properties of neural networks, which is a crucial feature in the design of neural networks. On the other hand, axonal signal transmission delays often occur in various neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Up to now, the stability analysis problem of neural networks with time-delay has been attracted a large amount of research interest and many sufficient conditions have been proposed to guarantee the asymptotic or exponential stability for the neural networks with various type of time delays such as constant, time-varying, or distributed. see for example [2], [4], [12-20] and [24], and the references therein.

Though the theoretical research on neural network has made great progress since it was born, but in many networks, such as in electronic neural networks, time delay can not be avoided. In fact, the stochastic perturbations can not be avoided either [6, 7, 10, 11,12,21]. On the other hand, the system is unavoidable uncertainty, which is due to the existence of modeling errors, can also destroy the stability of the neural networks. So it is very important to discuss the stability and robustness of cellular network against such error and fluctuation[8,12,14] .To overcome this difficulty, we will discuss the stability problem for a kind of stochastic interval cellular neural networks with time-delays (SICNND), and derive several exponential stability criteria for the cellular neural networks(SICNND).

This paper is organized as follows. In section 2, model description of the stochastic interval cellular neural networks with time-delays (SICNND), nomenclatures and lemma are given. In section 3, a kind of equivalent description of this stochastic interval delayed cellular neural networks and the idea for mean-square exponential stability are presented, and a set of some sufficient conditions is derived for the exponential stability of the stochastic interval cellular neural networks system. Finally, the conclusion is provided in section 4.

II. Problem Formulation and Preliminaries

Consider an SICNND state equation as follows:

$$dx(t)=[-Dx(t)+A_I\sigma(x(t))+B_I\sigma(x(t-\tau_1))]dt+f(t,x(t),x(t-\tau_2))dw(t) \qquad (1)$$

where $x(t)=(x_1(t),x_2(t),\cdots x_n(t))^T\in R^n$ is the state vector, and $D=diag(d_{ii})_{n\times n}\in R^{n\times n}, 0<d_{ii}, i=1,2,\cdots n$,

$$A_I=\{A\,|\,A=(a_{ij})_{n\times n}, p_{ij}\le a_{ij}\le q_{ij}, i,j=1,2,\cdots n\} \qquad (2)$$

$$B_I=\{B\,|\,B=(b_{ij})_{n\times n}, r_{ij}\le b_{ij}\le f_{ij}, i,j=1,2,\cdots n\} \qquad (3)$$

are $n\times n$ interval matrices. Where p_{ij} (r_{ij}) and q_{ij} (f_{ij}) are exactly known, so that $P=(p_{ij})_{n\times n}$, $Q=(q_{ij})_{n\times n}$, $R=(r_{ij})_{n\times n}$, $F=(f_{ij})_{n\times n}$ are constant matrices. $\tau_i\in(R_+,[0,\tau]), i=1,2, \tau>0$ are time lags. $\sigma(x)=(\sigma_1(x_1),\sigma_2(x_2),\cdots\sigma_n(x_n))^T$, satisfies $\sigma_i(x_i)=\frac{1}{2}(|x_i+1|-|x_i-1|)(1\le i\le n)$ $f(t,x(t),x(t-\tau_2))dw(t)$ denote random-perturbation. Where $w(t)=(w_1(t),w_2(t),\cdots w_m(t))^T(m\le n)$ is an m-dimensional Brownian motion which defined on a complete probability space (Ω,F,F_t,P) with a natural filtration $\{F_t\}_{t\ge0}$, (i.e., $F_t = \quad =\sigma\{w(s):0\le s\le t\}$), and $\sigma(\cdot)$ is non-linear activity function, $f:R_+\times R^n\times R^n\to R^{n\times m}$ is an local Lipschitz continuous which satisfy linear growth condition[3]. that is, for any $t\ge0$, there exists constants $c_i(d_i)\in[0,\infty)$ such that

$$\begin{cases}|\sigma_i(x_i)-\sigma_i(y_i)|\le(1\wedge c_i)|x_i-y_i|, & \forall x_i\ne y_i\\ |f(t,x,y)-f(t,\bar{x},\bar{y})|\le(1\wedge d_i)(|x-\bar{x}|+|y-\bar{y}|)\end{cases} \qquad (4)$$

where $(|x|\vee|\bar{x}|\vee|y|\vee|\bar{y}|)\le1$. And assume system (1) satisfy the initial condition $x(s)=\xi(s), -\tau\le s\le0, \xi\in L^2_{F_0}([-\tau,0],R^n)$, where $L^2_{F_0}([-\tau,0],R^n)$ is a stochastic process $\xi(s)$ with R^n value ($-\tau\le s\le0$), and $\xi(s)$ is F_0- measurable, $\int_{-\tau}^0 E|\xi(s)|^2ds<\infty$. We know that the system (1) has a unique solution by the paper [3], and it is denoted by $x(t,\xi)\hat{=}x(t)$, it is also square integrable.

In order to discuss the stability of system (1), assume, in the whole paper, $\forall t \in R_+ : f(t,0,0) \equiv 0$, and there exists $a_1, a_2 \geq 0$, such that $\forall (t,x,y) \in R_+ \times R^n \times R^n$, we have trace

$$f^T(t,x,y) f(t,x,y) \leq a_1 |x|^2 + a_2 |y|^2 \tag{5}$$

Particularly, $f^T(t,x(t),x(t-\tau_i)) f(t,x(t),x(t-\tau_i)) \leq a_1 |x(t)|^2 + a_2 |x(t-\tau_i)|^2$ on the other hand ,we can rewrite the interval matrices A_I and B_I in the following equivalent form:

$$A_I = A_0 + \Delta A \tag{6}$$

$$B_I = B_0 + \Delta B \tag{7}$$

where

$$A_0 = \tfrac{1}{2}(P+Q) \tag{8}$$

$$B_0 = \tfrac{1}{2}(R+F) \tag{9}$$

$$\Delta A = \sum_{i,j}^{n} \varepsilon_{ij} E_{ij}, E_{ij} = \tfrac{1}{2}(q_{ij} - p_{ij}) u_{ij}, u_{ij} = e_i e_j^T \tag{10}$$

$$\Delta B = \sum_{t,k}^{n} \varepsilon_{tk} E'_{tk}, E'_{tk} = \tfrac{1}{2}(f_{tk} - r_{tk}) u_{tk}, u_{tk} = e_t e_k^T \tag{11}$$

the uncertain parameters $|\varepsilon_{ij}| \leq 1$, e_i and e_j are unit vectors with only the ith and jth elements, being unity, respectively. u_{ij} is a matrix with all entries being zero except its (i, j) entry, which is unity. Also, we denote that (the case of ε_{ij}=1, i ,j=1,2,…n):

$$E = \sum_{i,j=1}^{n} E_{ij} = \tfrac{1}{2}(Q-P) \tag{12}$$

$$E' = \sum_{t,k=1}^{n} E'_{tk} = \tfrac{1}{2}(F-R) \tag{13}$$

And it are called as maximum perturbation matrix of A_I and B_I respectively. Obviously, the $E \geq 0 (E' \geq 0)$. Using the matrices $A_0(B_0)$ and $E(E')$, the interval matrix A_I and B_I can be represented by

$$A_I = [P,Q] = [A_0 - E, A_0 + E] = A_0 + \Delta A, |\Delta A| \leq E \tag{14}$$

$$B_I = [R,F] = [B_0 - E', B_0 + E'] = B_0 + \Delta B, |\Delta B| \leq E' \tag{15}$$

Where ΔA and ΔB are called as the bias matrices between A_I, A_0 and B_I, B_0, respectively. Also, we define

$$M_1 = (m_{ij})_{n\times n} = Q - A_0, \; N_1 = (n_{ij})_{n\times n} = F - B_0 \tag{16}$$

Obviously, the $M_1 \geq 0 (N_1 \geq 0)$,and we have

$$|\Delta A| \leq |M_1| = M_1 \leq E \;, \; |\Delta B| \leq |N_1| = N_1 \leq E' \tag{17}$$

The following notations and engagements are used throughout this paper:

Given a square complex matrix $A = (a_{ij}) \in C^{n\times n}$, $|A| = (|a_{ij}|)_{n\times n}$ denotes the modulus matrix of A, where $|a_{ij}|$ denotes the modulus (absolute value) of a_{ij}. For a real matrix $A = (a_{ij}) \in R^{n\times n}$ is nonnegative (positive) if its entries $a_{ij} \geq 0$ $(a_{ij} > 0)$ and $A \geq B \geq 0 (A > B > 0)$ means that the entriesof A and B satisfy $a_{ij} \geq b_{ij} \geq 0$ $(a_{ij} > b_{ij} > 0), i, j \in N = \{1,2,\cdots,n\}$, or $A - B \geq 0 (A - B > 0)$. $\rho(A)$ the spectral radius of matrix A, and I the identity matrix., $\lambda(A)$ denotes any eigenvalue of A. the operator norm or spectral norm of A is denoted by $\|A\|$.

Lemma 1[22] If A and B are quadratic matrices with nonnegative entries (some of the entries are positive) and $B \leq A$, then $\|B\|_p \leq \|A\|_p$ (p=1,2,∞) .

Note that the equivalence of the norm, which can be written as $\|A\|_p = \|A\|$ (p=1,2,∞) .

Associated (1), the following system is considered,

$$dx(t) = [-Dx(t) + A\sigma(x(t)) + B\sigma(x(t-\tau_1))]dt + f(t, x(t), x(t-\tau_2))dw(t) \tag{18}$$

$$A \in A_I, B \in B_I$$

III . Main Results

In view of the above section , a equivalent description of the systems (1) can be written as follows

$$dx(t) = [-Dx(t) + (A_0 + \Delta A)\sigma(x(t)) + (B_0 + \Delta B)\sigma(x(t-\tau_1))]dt \tag{19}$$

$$+ f(t, x(t), x(t-\tau_2))dw(t)$$

$$x(s) = \xi(s), -\tau \leq s \leq 0, \xi \in L^2_{F_0}([-\tau, 0], R^n)$$

Definition 2.1

The stochastic interval cellular neural networks with time-delays (1) is said to be robust mean-square exponential stable, if for any $\forall A \in A_I$, $\forall B \in B_I$, the system (18) is mean-square exponential stable or (19) is mean-square exponential stable.

Theorem 2.1

If there exists positively definite matrix G such that

$$\lambda_{\min}(GD + D^T G) > 2(\|GA_0\| + \|GM_1\|) + \tag{20}$$

$$+2(\|GB_0\| + \|GN_1\|)\sqrt{\lambda_{\max}(G)\lambda_{\min}^{-1}(G)} + \|G\|(a_1 + a_2 \lambda_{\min}^{-1}(G)\lambda_{\max}(G))$$

Then the trivial solution of systems (1) is robust mean-square exponential stable and almost surely exponential stable.

Proof For $\forall (t, x, y_1, y_2) \in R_+ \times R^n \times R^n \times R^n$, $\forall A \in A_I$, and $\forall B \in B_I$, (obviously $A = A_0 + \Delta A$, $B = B_0 + \Delta B$), define the lyapunov function as follows: $V(x) = x^T G x$

In terms of Lemma **1** and *Itô* inequality, we have the following

$$LV(x(t)) = V_x(x)(-Dx + A\sigma(x) + B\sigma(y_1))$$

$$+\tfrac{1}{2} trace f^T(t, x, y_2) V_{xx}(x) f(t, x, y_2)$$

$$= 2x^T G(-Dx + A\sigma(x) + B\sigma(y_1)) + trace f^T(t,x,y_2)Gf(t,x,y_2)$$

$$= 2x^T G(-Dx + (A_0+\Delta A)\sigma(x) + (B_0+\Delta B)\sigma(y_1))$$

$$+ trace f^T(t,x,y_2)Gf(t,x,y_2)$$

$$\le (-\lambda_{\min}(GD + D^T G) + 2\|G(A_0+\Delta A)\| + \|G(B_0+\Delta B)\|\beta + \|G\|a_1)|x|^2$$

$$+ \|G(B_0+\Delta B)\|\beta^{-1}|y_1|^2 + \|G\|a_2|y_2|^2$$

$$\le (-\lambda_{\min}(GD + D^T G) + 2(\|GA_0\| + \|GM_1\|) + (\|GB_0\| + \|GN_1\|)\beta + \|G\|a_1)|x|^2$$

$$+ (\|GB_0\| + \|GN_1\|)\beta^{-1}|y_1|^2 + \|G\|a_2|y_2|^2$$

$$\le -\lambda_1 V(x) + \lambda_2 V(y_1) + \lambda_3 V(y_2) \tag{21}$$

Where $\lambda_1 = (\lambda_{\min}(GD + D^T G) - 2(\|GA_0\| + \|GM_1\|) - (\|GB_0\| + \|GN_1\|)\beta + \|G\|a_1)\lambda_{\max}^{-1}(G)$

$$\lambda_2 = (\|GB_0\| + \|GN_1\|)\beta^{-1}\lambda_{\min}^{-1}(G),$$

$$\lambda_3 = \|G\|a_2\lambda_{\min}^{-1}(G), \beta = \sqrt{\lambda_{\min}^{-1}(G)\lambda_{\max}(G)}$$

By the condition (20), it is easy to show that , $\lambda_1 > \lambda_2 + \lambda_3$. Thus, by virtue of *Razumikhin-type* theorem in [3,5], we know the trivial solution of system (18) is mean-square exponential stable and almost surely exponential stable. Thus the theorem 2.1 is proved by the arbitrariness of A, B and Definition 2.1.

Theorem 2.2

If there exists positively definite matrix G such that

$$\lambda_{\min}(GD + D^T G) > 2(\|G(A_0 + B_0)\| + \|G(M_1 + N_1\|) + 2(\|GB_0\|$$

$$\|GN_1\|)\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)} + \|G\|(a_1 + a_2\lambda_{\min}^{-1}(G)\lambda_{\max}(G)) \tag{22}$$

Then the trivial solution of systems (1) is robust mean-square exponential stable and almost surely exponential stable. Where

$$H = [\tau(\|D\| + \|A_0\| + \|B_0\|) + \sqrt{\tau(\alpha_1 + \alpha_2)}]^2 \tag{23}$$

Proof Let $x(t)$ be a solution of systems(1), then $\forall i = 1,2$, by the *Hölder* inequality, we have

$$E|x(t) - x(t-\tau_i)|^2 \le (\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau$$

$$+ \sqrt{\tau(\alpha_1+\alpha_2)})(\frac{1}{\|D\|\tau}E\left|\int_{t-\tau_i}^t -Dx(s)ds\right|^2$$

$$+ \frac{1}{\|A_0+\Delta A\|\tau}E\left|\int_{t-\tau_i}^t (A_0+\Delta A)\sigma(x(s))ds\right|^2$$

$$+ \frac{1}{\|B_0+\Delta B\|\tau}E\left|\int_{t-\tau_i}^t (B_0+\Delta B)\sigma(x(s-\tau_i))ds\right|^2$$

$$+ \frac{1}{\sqrt{\tau(\alpha_1+\alpha_2)}}E\left|\int_{t-\tau_i}^t f dw(s))ds\right|^2),$$

$$+ \frac{1}{\|A_0+\Delta A\|\tau}E\left|\int_{t-\tau_i}^t (A_0+\Delta A)\sigma(x(s))ds\right|^2 +$$

$$+ \frac{1}{\|B_0+\Delta B\|\tau}E\left|\int_{t-\tau_i}^t (B_0+\Delta B)\sigma(x(s-\tau_i))ds\right|^2$$

$$+ \frac{1}{\sqrt{\tau(\alpha_1+\alpha_2)}}E\left|\int_{t-\tau_i}^t f dw(s))ds\right|^2),$$

By employing the *Hölder* inequality go on, we can get

$$E|x(t) - x(t-\tau_i)|^2 \le (\|D\|\tau + \|A_0 + \Delta A\|\tau$$

$$\|B_0 + \Delta B\|\tau + \sqrt{\tau(\alpha_1+\alpha_2)})(\frac{1}{\|D\|\tau}\|D\|^2\tau^2 \sup_{-2\tau\le\theta\le 0} E|x(t+\theta)|^2$$

$$+ \frac{1}{\|A_0+\Delta A\|\tau}\|A_0+\Delta A\|^2\tau^2 \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2$$

$$+ \frac{1}{\|B_0+\Delta B\|\tau}\|B_0+\Delta B\|^2\tau^2 \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2 \tag{24}$$

$$+ \frac{1}{\sqrt{\tau(\alpha_1+\alpha_2)}}E\left|\int_{t-\tau_i}^t f dw(s))ds\right|^2),$$

By virtue of lemma2.3 in [7] and note that (5), we have

$$E|x(t) - x(t-\tau_i)|^2 \le (\|D\|\tau + \|A_0 + \Delta A\|\tau$$

$$+ \|B_0 + \Delta B\|\tau + \sqrt{\tau(\alpha_1+\alpha_2)})\|D\|\tau \sup_{-2\tau\le\theta\le 0} E|x(t+\theta)|^2$$

$$+ \|A_0 + \Delta A\|\tau \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2 +$$

$$+ \|B_0 + \Delta B\|\tau \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2$$

$$+ \frac{1}{\sqrt{\tau(\alpha_1+\alpha_2)}}\tau(\alpha_1+\alpha_2) \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2$$

$$\le (\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau + \sqrt{\tau(\alpha_1+\alpha_2)})$$

$$+ (\|D\|\tau \sup_{-2\tau\le\theta\le 0} E|x(t+\theta)|^2 + \|A_0 + \Delta A\|\tau \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2$$

$$+ \|B_0 + \Delta B\|\tau \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2$$

$$+ \sqrt{\tau(\alpha_1+\alpha_2)} \sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2)$$

$$\le (\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau$$

$$+ \sqrt{\tau(\alpha_1+\alpha_2)})(\|D\|\tau + \|A_0 + \Delta A\|\tau$$

$$+ \|B_0 + \Delta B\|\tau + \sqrt{\tau(\alpha_1+\alpha_2)})(\sup_{-\tau\le\theta\le 0} E|(x(t+\theta)|^2)$$

$$\le (\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau +$$

$$+ \sqrt{\tau(\alpha_1+\alpha_2)})(\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau +$$

$$+ \sqrt{\tau(\alpha_1+\alpha_2)})(\sup_{-\tau\le\theta\le 0} E|(x(t+\theta)|^2)$$

$$= [\|D\|\tau + \|A_0 + \Delta A\|\tau + \|B_0 + \Delta B\|\tau + \sqrt{\tau(\alpha_1+\alpha_2)}]^2 \cdot$$

$$\cdot(\sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2)$$

$$\le [(\|D\| + \|A_0\| + \|B_0\| + \|M_1\| + \|N_1\|)\tau + \sqrt{\tau(\alpha_1+\alpha_2)}]^2 \cdot$$

$$\cdot(\sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2),$$

$$i.e., E|x(t) - x(t-\tau_i)|^2 \le$$

$$\le [(\|D\| + \|A_0\| + \|B_0\| + \|M_1\| + \|N_1\|)\tau + \sqrt{\tau(\alpha_1+\alpha_2)}]^2 \cdot$$

$$\cdot(\sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2)$$

$$\hat{=} H(\sup_{-2\tau\le\theta\le 0} E|(x(t+\theta)|^2), \cdots\cdots\cdots\cdots\cdots\cdots (25)$$

Now, For $\forall A \in A_I$, $\forall B \in B_I$ (obviously $A = A_0 + \Delta A$, $B = B_0 + \Delta B$), define lyapunov function as follows: $V(x) = x^T Gx$ by using the *Itô* inequality, and notice that (5),we have

$$LV(x(t)) = 2x^T(t)G(-Dx(t)) + (A+B)\sigma(x(t)) +$$

$$+2x^T(t)GB(\sigma(x(t-\tau_i)) - \sigma(x(t))) + trace f^T Gf$$

$$= 2x^T(t)G(-Dx(t)) + (A_0 + \Delta A + B_0 + \Delta B)\sigma(x(t))$$

$$+2x^T(t)G(B_0 + \Delta B)(\sigma(x(t-\tau_i)) - \sigma(x(t))) + trace f^T Gf$$

$$\le -[\lambda_{\min}(GD + D^T G) - 2\|G(A_0 + B_0)\| -$$

$$-\|GB_0\|\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)} - \|G\|\alpha_1]|x(t)|^2$$

$$+\frac{\|GB_0\|}{\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)}}|x(t-\tau_1) - x(t)|^2 + \|G\|\alpha_2|x(t-\tau_2)|^2 \qquad (26)$$

by the (22) we have

$$\lambda_{\min}(GD + D^T G) > 2\|G(A_0 + B_0)\| +$$

$$+2\|GB_0\|\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)} + \|G\|(a_1 + a_2\lambda_{\min}^{-1}(G)\lambda_{\max}(G)), \qquad (27)$$

Thus, by virtue of (25),(27), from the (26) we can get

$$ELV(x(t)) \le -[\lambda_{\min}(GD + D^T G) - 2\|G(A_0 + B_0)\|$$

$$-\|GB_0\|\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)} - \|G\|\alpha_1]\lambda_{\max}^{-1}(G)EV(x(t)) \qquad (28)$$

$$+(\frac{\|GB_0\|H}{\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)}} + \|G\|\alpha_2)\lambda_{\min}^{-1}(G)\sup_{-2\tau\le\theta\le 0} V(x(t+\theta))$$

From the (27),it is easy to see that there is exist $q > 1$ such that

$$-\lambda \hat{=} -(\lambda_{\min}(GD + D^T G) - 2\|G(A_0 + B_0)\| - \|GB_0\|\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)}$$

$$-\|G\|\alpha_1]\lambda_{\max}^{-1}(G) + (\frac{\|GB_0\|H}{\sqrt{H\lambda_{\max}(G)\lambda_{\min}^{-1}(G)}} + \|G\|\alpha_2)\lambda_{\min}^{-1}(G)q < 0. \qquad (29)$$

thus, as the inequality $EV(x(t+\theta)) < qEV(x(t)), -2\tau \le \theta \le 0$ holds, by the (28),(29), we can obtain

$$ELV(x(t)) \le -\lambda EV(x(t)), \qquad (30)$$

Thus, by virtue of *Razumikhin-type* theorem in [3,5], we know that the trivial solution of system (18) is robust mean-square exponential stable and almost surely exponential stable. Thus the Theorem 2.2 is proved by the arbitrariness of A, B and Definition 2.1.

IV. Conclusions

The above we have discussed the robust mean-square exponential stability problem for a class of stochastic interval cellular neural networks with time-delays. by means of the *Itô* formula, *Razumikhin* theorems, *Lyapunov* function and norm inequalities, several sufficient conditions are obtained which guarantee the mean-square exponential stability of the stochastic interval cellular neural networks with time-delays. and some recent results reported in the literatures are generalized.

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