The Effect of Frequency Offset on Network Coding with 4FSK Modulation*

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Abstract - We evaluate a scheme of single-relay network coding based on 4FSK (4 Frequency Shift Keying) modulation. Firstly, a loose upper bound BER (Bit Error Rate) of 4FSK is derived in the presence of frequency offset in the direct link. Secondly, 4FSK modulation is utilized in the three-slot two-way relay network and the BER of the transmission scheme is analysed and simulated in the case of the frequency offset.

Index Terms - Network coding, BFSK, non-coherent demodulation, frequency offset.

I . Introduction

The theory of network coding was put forward by Rudolf Ahlswede, etc. in 2000[1]. It was originally used to improve the throughput of wired network. Shengli Zhang in 2006, proposed the concept of the physical layer network coding [2]. In the two-way relay network, the two terminals convey the bits at the same time. The relay receives a combination of both modulated bits in the first slot, which are conveyed to the two terminals in the second slot, each terminal can recover the information from the other terminal by subtracting its own bits from the received signals. In this way the system further reduces the required transmission slots and improves the capacity of system. In the case of three orthogonal slots in the design of two-way relaying[3], two sources A and B, respectively utilizes single BPSK modulation on the same sinusoidal carrier frequency signal and the cosine carrier frequency signal and this hybrid modulation can bring about 3dB than the traditional two-way relay network coding. The joint design of the physical layer network coding and channel coding can enhance the overall reliability of the system [4] [5]. In [6] [7], the scheme with 2-slot network coding uses frequency-shift keying (FSK) modulation and operates in non-coherent mode. By selecting the appropriate channel coding rate, two-way relay network with 2-slot transmission can achieve better performance than that with 3-slot transmission. They also consider different receivers for different channel conditions.

Most of the papers mentioned above, the wireless network coding schemes typically use BPSK modulation. Moreover perfect frequency and phase synchronization are assumed. However, in the real communication, the perfect frequency and phase synchronization condition is not always satisfied. For example, in [5], the system of wireless network coding using BPSK supposes that the receiver has perfect frequency synchronization. Therefore, it is necessary for us to do further research on impact of frequency offset on the wireless network coding. We focus on a single two-way relay network. We first derive loose upper bound BER of 4FSK non-coherent demodulation in the presence of frequency offset. Secondly, we further consider a single relay, two-way relay system which uses 4FSK modulation.

II. The derivation of the upper bound BER of 4FSK in the presence of frequency offset

In the section, we only consider the 4FSK transmission in the direct link. For 4FSK modulation signal, the receiver can perform non-coherent demodulation, which can be done in lack of phase synchronization between a transmitter and a receiver. The model for 4FSK modulation and demodulation is discussed in [8] [9], where each I-branch and Q-branch in the receiver comprises four carrier frequency sub-branches. Here we assume only one of the four sub-branches will process the transmitted signal plus noise and the remaining sub-branches only process pure noises. In addition, the carrier frequency, the phase and frequency offset between the transmitter and the receiver is f , θ and Δf respectively. The signal is represented (without noise)

$$s(t) = A\cos(2\pi\left(f_1 + \Delta f_1\right)t + \theta_{11}) \tag{1}$$

The signal process at I-branch of the receiver is

$$\begin{aligned}&\frac{2}{T}\int_0^T \cos 2\pi f_i t \times s\left(t\right) dt \\ &= \frac{A\sin 2\pi\Delta fT}{2\pi\Delta fT}\cos\theta - \frac{A(1-\cos 2\pi\Delta fT)}{2\pi\Delta fT}\sin\theta \\ &= A_{jI}\end{aligned} \tag{2}$$

For Q-branch, we have

* This work was supported in part by National Mobile Communications Research Lab-oratory, Southeast University(2011D17, 2011D05), Basic Research Program of Jiangsu Province (Natural Science Foundation) (BK2011756), University Natural Science Research Program of Jiangsu Province (11KJB510018, 11KJB510016), National Natural Science Foundation of China (61271240, 61001077 and 61201164), Specialized Research Fund for the Doctoral Program of Higher Education (20113223120002), Jiangsu postdoctoral program (1201014c) and NUPT Foundation (NY211040).

$$\begin{aligned}&\frac{2}{T}\int_0^T \sin 2\pi ft\times s(t)\,dt\\&=\frac{A(\cos 2\pi\Delta fT-1)}{2\pi\Delta fT}\cos\theta-\frac{A\sin 2\pi\Delta fT}{2\pi\Delta fT}\sin\theta\\&=A_{jQ}\end{aligned} \quad (3)$$

Then, we have the combined I-branch and Q-branch value

$$\begin{aligned}&A^2{}_{jI}+A^2{}_{jQ}\\&=\left(\frac{A\sin 2\pi\Delta fT}{2\pi\Delta fT}\right)^2+\left(\frac{A(\cos 2\pi\Delta fT-1)}{2\pi\Delta fT}\right)^2\\&=A^2\sin c^2(\Delta fT)\end{aligned} \quad (4)$$

Referred to [9], the decision signal to each sub-branch can be expressed as, respectively

$$T_1(t)=\sqrt{\left(A_{jI}+n_{1I}(t)\right)^2+\left(A_{jQ}+n_{1Q}(t)\right)^2} \quad (5)$$

$$T_2(t)=\sqrt{\left(n_{2I}(t)\right)^2+\left(n_{2Q}(t)\right)^2} \quad (6)$$

$$T_3(t)=\sqrt{\left(n_{3I}(t)\right)^2+\left(n_{3Q}(t)\right)^2} \quad (7)$$

$$T_4(t)=\sqrt{\left(n_{4I}(t)\right)^2+\left(n_{4Q}(t)\right)^2} \quad (8)$$

where $n_{iI}(t),\ n_{iQ}(t), i=1,2,3,4$ are Gaussian random variables with zero mean and variance $\sigma_n{}^2$. We denote the sampling value of envelope $T_i(t)$ as T_i which obeys a generalized Rayleigh distribution and denote x as the normalized frequency offset, then we have

$$\begin{aligned}f(T_1)&=\frac{T_1}{\sigma^2{}_n}I_0\left(\frac{AT_1\sin cx}{\sigma^2{}_n}\right)\\&\times e^{-\frac{T_1^2+(A\sin cx)^2}{2\sigma^2{}_n}}\end{aligned} \quad (9)$$

$$f(T_j)=\frac{T_j}{\sigma^2{}_n}e^{-\frac{T_j^2}{2\sigma^2{}_n}},\text{(j=2,3,4)} \quad (10)$$

The probability that three noise-only sub-branch does not exceed a certain threshold is

$$p=[1-p(l)]^3 \quad (11)$$

where $p(l)$ represents the probability that noise does not exceed a certain threshold, i.e.,

$$p(l)=\int_l^\infty \frac{V}{\sigma^2{}_n}e^{\frac{-V^2}{2\sigma^2{}_n}}dV \quad (12)$$

If the three sub-branches which contain only noise not exceed the threshold, the judgment process will not go wrong. If there is more than one sub-branch, it will cause a judgment error, the error probability is

$$\begin{aligned}p_e(l)&=1-[1-p(l)]^3\\&=1-[1-e^{-l^2/2\sigma^2{}_n}]^3\\&=\sum_{n=0}^{3}(-1)^n\binom{3}{n}e^{-nl^2/2\sigma_n{}^2}\end{aligned} \quad (13)$$

where $\binom{3}{n}$ is the binomial expansion coefficient.

For simplicity, we omit the subscript of T_1. Referred to [9], the sampling value T can be represented in the presence of the frequency offset

$$\begin{aligned}p(T)&=\frac{T}{\sigma^2{}_n}I_0\left(\frac{AT\sin cx}{\sigma^2{}_n}\right)\\&\times e^{-\frac{T^2+(A\sin cx)^2}{2\sigma^2{}_n}}\quad(T\ge 0)\end{aligned} \quad (14)$$

where T is the sampling value of the received signal plus noise, which equals the threshold of (2). Therefore, the probability of an error is

$$p_e=\int_0^\infty p_e(l)\,p(l)dl \quad (15)$$

Substituting (13) and (14) into (15) , the averaged symbol error probability is

$$\begin{aligned}p_e&=e^{-\frac{(A\sin c(x))^2}{2\sigma_n{}^2}}\sum_{n=1}^{3}(-1)^{n-1}\binom{3}{n}\\&\times\int_0^\infty \frac{l}{\sigma_n{}^2}I_0\left(\frac{A\sin c(x)}{\sigma_n{}^2}\right)e^{-(1+n)l^2/2\sigma_n{}^2}dl\end{aligned} \quad (16)$$

Due to $\int_0^\infty l\cdot I_0(al)e^{-(a^2+l^2)/2}dl=1$ in [8], we can get from (15)

$$p_e=\sum_{n=1}^{3}(-1)^{n-1}\binom{3}{n}\frac{1}{n+1}e^{-(nA\sin c(x))^2/2(n+1)\sigma^2{}_n} \quad (17)$$

Then the bound of the symbol error probability is

$$p_e\le\frac{3}{2}e^{-(A\sin c(x))^2/4\sigma_n{}^2}=\frac{3}{2}e^{-r\sin c^2(x)/2} \quad (18)$$

Due to $r=A^2/2\sigma^2{}_n$, the bit error probability is

$$p_b\le\frac{2^{k-1}}{2^k-1}p_e=e^{-r\sin c^2(x)/2} \quad (19)$$

III. The model of two-way relay transmission and the analysis of BER

A. The three slot two-way relay network model

The sources exchange information in three slots in the studied two-way relay transmission. In the first slot, the source terminal A transmits b_1 messages by using 4FSK modulation. The receiver of Relay R estimate $\hat{b}_1$ through non-coherent demodulation mode. In the second slot, the relay R obtains estimate $\hat{b}_2$ from the source terminal B. Then R encodes $\hat{b}_1$ and $\hat{b}_2$ to output b, i.e. $b=\hat{b}_1\oplus\hat{b}_2$. In the third slot, R broadcasts b to both A and B by using 4FSK modulation and the source terminals A and B can obtain estimate $\hat{b}$ through non-coherent demodulation. After A receives $\hat{b}$, it can recover the messages sent by B using local messages by the means of $u_1=b_1\oplus\hat{b}$. In the same way, B can get the messages sent by A by the means of $u_2=b_2\oplus\hat{b}$. At present most of the two-way relay network coding based on the frequency shift keying articles are in frequency synchronization. In the following sub-section, we focus on the two-way relay network coding in the presence of frequency offset.

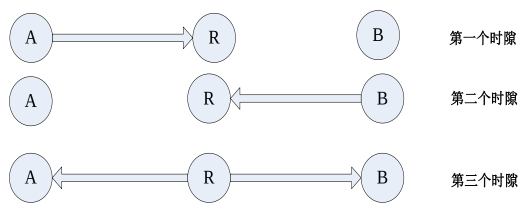

Fig. 1 The three slot two-way relay network model

B. BER Analysis of Three-Slot Two-Way Relaying

We will analyse the BER of the three time slots two-way relay system in the case of frequency offset. Suppose that the normalized frequency offset in the link from A to R is x_{AR}. Assumption frequency offset from B to R in the process of signal transmission in the system is x_{BR} (x_{BR} is normalized frequency offset). In the same way, the frequency offset from R to A is x_{AR} and the frequency offset from R to B is x_{RB}. For simplified analysis, we x_{RB} and x_{BR} is equal and this is same to x_{AR} and x_{RA}. In the first slot, according to (19), the bit error rate of received data from A in the relay R is

$$p_{AR}\le e^{-r\sin c^2(x_{AR})/2} \quad (20)$$

In the second slot, the bit error rate of received data from B in the relay R is

$$p_{BR}\le e^{-r\sin c^2(x_{BR})/2} \quad (21)$$

Hence relay's bit error rate of received data is

$$p_R=p_{AR}\times\left(1-p_{BR}\right)+p_{BR}\left(1-p_{AR}\right) \quad (22)$$

In the third slot, the bit error rate from R to A is

$$p_{RA}\le e^{-r\sin c^2(x_{RA})/2} \quad (23)$$

The bit error rate from R to B is

$$p_{RB}\le e^{-r\sin c^2(x_{RB})/2} \quad (24)$$

The bit error rate of received data in A is

$$p_A=p_R\times\left(1-p_{RA}\right)+p_{RA}\left(1-p_R\right) \quad (25)$$

From the (24), we observe that signal frequency offset makes an impact on the system's BER, and the greater the offset, the greater the bit error rate of the system.

IV . The simulation results

The simulation experiments under the normalized frequency offset at 0,1/8,1/6 and 1/4 are performed. From Fig. 2, it is observed that, as the frequency offset increases, 4FSK non-coherent demodulation error rate increases, and the corresponding upper bound on the error rate also increases. The three red curves (marked with "o", "+", " · " and "□ " ,respectively) are the upper bound of BER in the direct link as well as the red curve without frequency offset. We also observe from this figure that, when the bit error rate is at 10^{-5}, the scheme with frequency offset of 1/4 needs more 2 dB SNR than that of the system with no offset in the direct link.

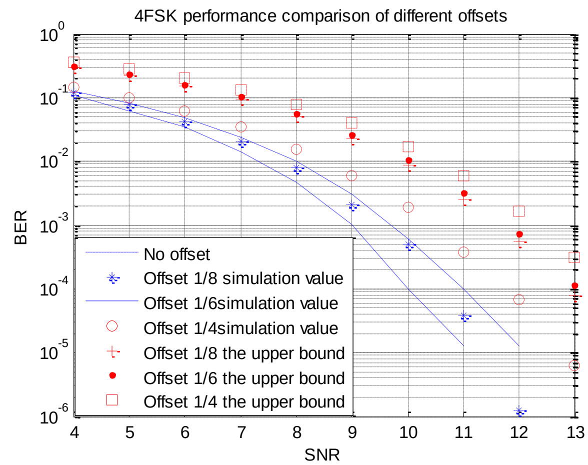

Fig. 2 BER of 4FSK noncoherent demodulation with different offsets.

Next, we do experiments in three-slot two-way relay sche me with the 4FSK modulation. Under the Gaussian channel m odel, we continue the simulation experiments with the frequen cy offsets of 0,1/8,1/6 and 1/4. At the receiver of terminal B, t wo cases i.e., no frequency offset (in Fig. 3) and 1/6 offset (in Fig. 4) are considered. As seen from the Figure 3 and Figure 4, when the bit error rate is at 10^{-4}, the system with frequency of fset of 1/4 needs more 2 dB SNR than that of the system with n o offset.

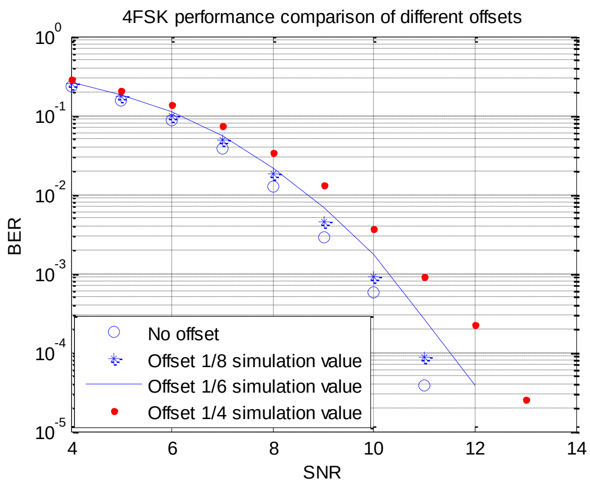

Fig. 3 BER of three slots network coding (only the terminal B without offset).

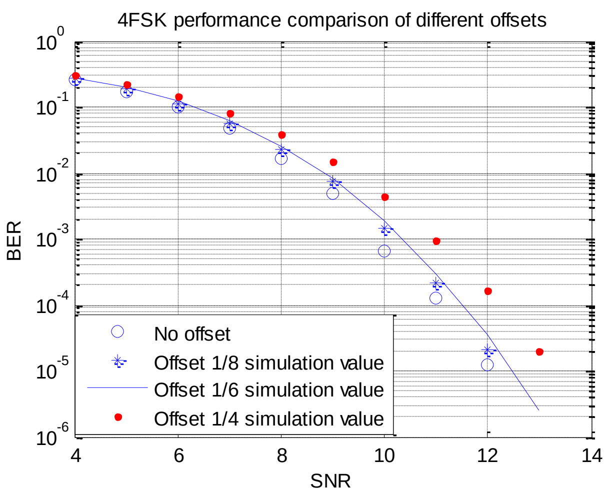

Fig 4 BER of three slots network coding (only the terminal B with 1/6 offset)

V. Conclusions

We consider the transmission system containing frequency offset in 4FSK non-coherent demodulation in additive Gaussian channel. Firstly, a loose upper bound BER of 4FSK scheme is deduced with frequency offset in direct link, and the effectiveness of the formula is verified by computer simulation. Secondly, 4FSK modulation is applied to three-slot two-way relay network. Simulation results show that when the error rate reaches 10^{-4}, the normalized frequency offset 1/4 will lead about 2 dB SNR losses in the three-slot network coding strategy. Since only the frequency offset of three-slot two-way relay scheme is considered in this work, the influence of frequency offset on two-slot two-way relay scheme remains to be done in further research in the future.

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