Since $V(t^*+\bar{T}) \geq V_L$, repeat above process we have $V(t) \geq V_L$ for all $t > \bar{t}$.

On the other hand, if $t^* \neq n_1 T, n_1 \in N$, it is very similar to prove that there exists a $\tilde{t} > 0$ such that $V(t) \geq V_L$ for all $t > \bar{t}$.

And as the page is limited, we omit this process.

That is to say, $\liminf_{t\to\infty} V(t) \geq V_L$ holds, combined with Lemma 3, the virus population is permanent.

V. Numeric Simulations and discussions

In this paper, we have analyzed the global attractivity of the virus-free periodic solution of a viral infection SVI model with lytic immune response and periodic medication strategy. And we obtained an important critical value $\Re_0$ successfully, it is shown that the VFPS is globally attractive and the virus will be eradicated finally when $\Re_0 < 1$ in section 3, while it is shown that the virus will be permanent if $\Re_0 > 1$ in section 4.

Thus, in order to verify above theoretical results, we will give some numerical simulations in the following.

For system (2) if we choose

$$r = 0.8, d = 0.1, \alpha = 0.3, \beta = 0.3, p = 0.8, q = 1.0, b = 0.1,$$
$$c = 0.6, T = 1, \delta_1 = 0.2, \delta_2 = 0.4, \delta_3 = 0.1$$

with initial conditions

$$S_0 = 0.5, V_0 = 0.3, I_0 = 0.2 \quad (26)$$

and by a direct computation we have

$$\Re_0 = \frac{d\left(e^{dT} - (1-\delta_1\right)}{r\beta\delta_1(e^{dT}-1)}\left(d\ln(1-\delta_2) + T(\beta r - \alpha d)\right) \approx 0.9607 < 1$$

One hand, by Theorem 1, the VFPS $(S_0(t), 0, 0)$ of system (2) is globally attractive; On the other hand, by Maple software, we can plot the time series diagram for $(S(t), V(t), I(t))$ as Fig.1, and one can see that both the virus population and the immune response (CD4+T cells) will be eventually eradicated, but the susceptible host cells S(t) can keep permanent.

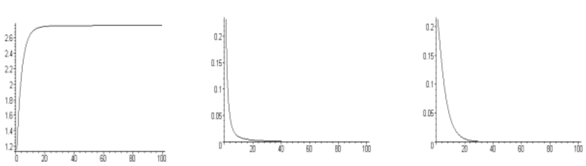

Fig.1 Time series diagram for $(S(t), V(t), I(t))$ with T=1.

If we only prolong the medication period, other parameters keep the same, such that $T = 2$. By a simple computation we obtain

$$\Re_0 = \frac{d\left(e^{dT} - (1-\delta_1\right)}{r\beta\delta_1(e^{dT}-1)}\left(d\ln(1-\delta_2) + T(\beta r - \alpha d)\right) \approx 1.4629 > 1$$

One hand, by Theorem 2, the virus population will be permanent; on the other hand, by Maple software, we can plot the time series diagram for $(S(t), V(t), I(t))$ as Fig.2, and one can see that both the virus population and the immune response (CD4+T cells) will be permanent.

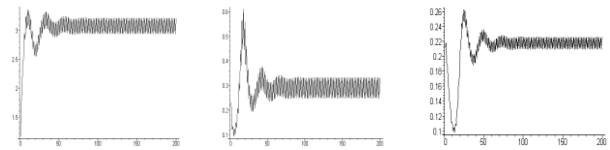

Fig.2 Time series diagram for $(S(t), V(t), I(t))$ with T=2, δ2=0.4.

Above two examples show that if we intend to eliminate the virus population effectively, we must choose suitable medication period, if the medication period is too long, the medication strategy might be failed. However, intuitively speaking, as long as we increase the intensity of each medication, i.e, increase the parameter $\delta_i (i = 1, 2, 3)$, we might also be able to eradicate the viral population, but in fact is it right? And if we choose $\delta_2 = 0.7 > 0.4$ and other parameters keep the same, then

$$\Re_0 = \frac{d\left(e^{dT} - (1-\delta_1\right)}{r\beta\delta_1(e^{dT}-1)}\left(d\ln(1-\delta_2) + T(\beta r - \alpha d)\right) \approx 1.1880 > 1$$

Also, the virus population will be permanent by Theorem 1; and we can also plot the time series diagram as Fig.3 by Maple software. From Fig.3, we can see that the immune response (CD4+T cells) will be eventually eradicated while the virus population will still be permanent. And this phenomena tell us excessive medication strategy will not kill the virus effectively instead of eradicating the immune response.

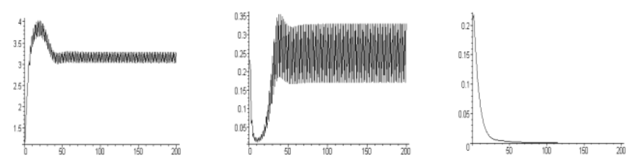

Fig.3 Time series diagram for $(S(t), V(t), I(t))$ with T=2, δ2=0.7.

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