The two variable (G'/G,1/G) -expansion method for finding exact traveling wave solutions of the (3+1) - dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama equation

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Abstract - The two variable $(G'/G,1/G)$-expansion method is employed to construct exact traveling wave solutions with parameters of the (3 + 1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama (YTSF) equation. When the parameters are replaced by special values, the well-known solitary wave solutions of this equation rediscovered from the traveling waves. This method can be thought of as the generalization of the well-known original (G'/G)-expansion method proposed by M. Wang et al. It is shown that the two variable $(G'/G,1/G)$-expansion method provides a more powerful mathematical tool for solving many other nonlinear PDEs in mathematical physics.

Index Terms - The two variable $(G'/G,1/G)$-expansion method ; The (3 + 1)-dimensional potential YTSF equation ; Exact traveling wave solutions; Solitary wave solutions.

1. Introduction

In the recent years, investigations of exact solutions to nonlinear partial differential equations (NPDEs) play an important role in the study of nonlinear physical phenomena. Many powerful methods have been presented, such as the inverse scattering method [1], the Hirota bilinear transform [2], the truncated painleve expansion method [3-6], the Backlund transsform method [7,8], the exp-function method [9-13], the tanh function method [14-17], the Jacobi elliptic function expansion method [18-20], the (G'/G)-expansion method [21-30], the modified (G'/G)-expansion method [31], the $(G'/G,1/G)$-expansion method [32,33], the first integral method [34] and so on. The key idea of the one variable (G'/G) -expansion method is that the exact solutions of nonlinear PDEs can be expressed by a polynomial in one variable (G'/G) in which $G=G(\xi)$ satisfies the second order linear ODE $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0,$ where λ and μ are constants and $'=d/d\xi$. The key idea of the two variable $(G'/G,1/G)$-expansion method is that the exact traveling wave solutions of nonlinear PDEs can be expressed by a polynomial in the two variables (G'/G) and $(1/G)$ in which $G=G(\xi)$ satisfies the second order linear ordinary differential equation $G''(\xi)+\lambda G(\xi)=\mu$, where λ and μ are constants. The degree of this polynomial can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in the given nonlinear PDEs. The coefficients of this polynomial can be obtained by solving a set of algebraic equations resulted from the process of using this method. Recently, Li et al [32], have applied the $(G'/G,1/G)$-expansion method and determined the exact solutions of Zakharov equations, while Zayed et al [33] have used this method to find the exact solutions of the combined KdV-mKdV equation.

The objective of this paper is to apply the two variable $(G'/G,1/G)$-expansion method to find the exact traveling wave solutions of the following nonlinear (3+1)-dimensional potential YTSF equation [35]:

$$-4u_{xt}+u_{xxxz}+4u_x u_{xz}+2u_z u_{xx}+3u_{yy}=0. \qquad (1.1)$$

This equation is a potential-type counterpart of the (3+1)-dimensional nonlinear equation

$$[-4v_t+\Phi(v)v_z]_x+3v_{yy}=0,$$
$$\Phi=\partial_x^2+4v+2v_x+2v_x\partial_x^{-1}, \qquad (1.2)$$

introduced by Yu el al [35], while making the (3+1)-dimensional generalization from the (2 + 1)- dimensional Calogero-Bogoyavlenkii-Schiff equation [36]:

$$-4v_t+\Phi(v)v_z=0,\ \Phi=\partial_x^2+4v+2v_x\partial_x^{-1}, \qquad (1.3)$$

as did for the KP equation from the KdV equation. Taking $v=u_x$ the equation (1.2) transforms into the potential-YTSF equation (1.1). We also remark that the equation (1.1) itself becomes the potential KP equation if $z=x$ and reduces to the potential KdV equation while further taking $u_y=0.$ Therefore, various applications of the KP and KdV equations show great potential for applications of (1.1) in the physical sciences. Recently, Zayed [29] have discussed this equation using the (G'/G)-expansion method. The rest of this paper is organized as follows : In Sec. 2, we give the description of the two variable $(G'/G,1/G)$-expansion method. In Secs. 3, we apply this method to solve Eq. (1.1). In Sec. 4, some conclusions are given.

2. Describtion of the Two Variable (G'/G,1/G) -Expansion Method

Before, we describe the main steps of this method, we need the following remarks (see [32, 33]):

Remark 1. If we consider the second order linear ODE:

$$G''(\xi)+\lambda G(\xi)=\mu \tag{2.1}$$

and set $\phi=\frac{G'}{G}$, $\psi=\frac{1}{G}$, then we get

$$\phi'=-\phi^2+\mu\psi-\lambda,\quad \psi'=-\phi\psi. \tag{2.2}$$

Remark 2. $\lambda<0$, then the general solutions of Eq. (2.1) has the form:

$$G(\xi)=A_1\sinh(\xi\sqrt{-\lambda})+A_2\cosh(\xi\sqrt{-\lambda})+\frac{\mu}{\lambda}, \tag{2.3}$$

where A_1 and A_2 are arbitrary constants. Consequently, we have

$$\psi^2=\frac{-\lambda}{\lambda^2\sigma+\mu^2}(\phi^2-2\mu\psi+\lambda), \tag{2.4}$$

where $\sigma=A_1^2-A_2^2$.

Remark 3. If $\lambda>0$, then the general solutions of Eq. (2.1) has the form:

$$G(\xi)=A_1\sin(\xi\sqrt{\lambda})+A_2\cos(\xi\sqrt{\lambda})+\frac{\mu}{\lambda}, \tag{2.5}$$

and hence

$$\psi^2=\frac{\lambda}{\lambda^2\sigma-\mu^2}(\phi^2-2\mu\psi+\lambda), \tag{2.6}$$

where $\sigma=A_1^2+A_2^2$.

Remark 4. If $\lambda=0$, then the general solutions of Eq. (2.1) has the form:

$$G(\xi)=\frac{\mu}{2}\xi^2+A_1\xi+A_2, \tag{2.7}$$

and hence

$$\psi^2=\frac{1}{A_1^2-2\mu A_2}(\phi^2-2\mu\psi). \tag{2.8}$$

Suppose we have the following nonlinear evolution equation

$$F(u,u_t,u_x,u_y,u_z,u_{xx},...)=0, \tag{2.9}$$

where F is a polynomial in $u(x,y,z,t)$ and its partial derivatives. In the following, we give the main steps of the $(G'/G,1/G)$-expansion method [32, 33]:

Step 1.The traveling wave transformation

$$u(x,y,z,t)=u(\xi),\qquad \xi=x+y+z-\omega t, \tag{2.10}$$

where ω is a constant, reduces Eq.(2.9) to an ODE in the form:

$$P(u,u',u'',...)=0, \tag{2.11}$$

where P is a polynomial of $u(\xi)$ and its total derivatives with respect to ξ

Step 2. Assuming that the solution of Eq.(2.11) can be expressed by a polynomial in the two variables ϕ and ψ as follows:

$$u(\xi)=\sum_{i=0}^{N}a_i\phi^i+\sum_{i=1}^{N}b_i\phi^{i-1}\psi, \tag{2.12}$$

where $a_i\,(i=0,1,...,N)$ and $b_i\,(i=0,1,...,N)$ are constants to be determined later.

Step 3. Determine the positive integer N in Eq.(2.12) by using the homogeneous balance between the highest-order derivatives and the nonlinear terms in Eq.(2.11).

Step 4. Substitute Eq.(2.12) into Eq.(2.11) along with (2.2) and (2.4), the left- hand side Eq.(2.11) can be converted into a polynomial in ϕ and ψ, in which the degree of ψ is not longer than 1. Equating each coefficients of this polynomial to zero, yields a system of algebraic equations which can be solved by using the Maple or Mathematica to get the values of a_i, b_i, ω, μ, A_1, A_2, and λ where $\lambda<0$.

Step 5. Similar to step 4, substitute Eq.(2.12) into Eq.(2.11) along with (2.2) and (2.6) for $\lambda>0$, (or (2.2) and (2.8) for $\lambda=0$), we obtain the exact solutions of Eq.(2.11) expressed by trigonometric functions (or by rational functions) respectively..

3. Applications

In this section, we will apply the method described in Sec.2 to find the exact traveling wave solutions of the nonlinear (3+1)-dimensional potential YTSF equation (1.1). To this end, we see that the traveling wave transformation (2.10) permits us converting Eq.(1.1) into the following ODE:

$$u'''+3u'^2+(4\omega+3)u'=0, \tag{3.1}$$

with zero constant of integration. By balancing between u''' with u'^2 in Eq. (3.1) we get $N = 1$. Consequently, we get

$$u(\xi) = a_0 + a_1\phi + b_1\psi, \qquad (3.2)$$

where a_0, a_1 and b_1 are constants to be determined later.
There are three cases to be discussed as follows:

Case 1. Hyperbolic function solutions ($\lambda < 0$)

If $\lambda < 0$, substituting (3.2) into (3.1) and using (2.2) and (2.4), the left-hand side of Eq. (3.1) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to be zero, yields a system of algebraic equations in a_0, a_1, b_1, μ, λ, and ω as follows:

$$\phi^4 : -6a_1 + 3a_1^2 - \frac{3\lambda b_1^2}{\lambda^2\sigma + \mu^2} = 0,$$

$$\phi^3\psi : -6b_1 + 6a_1b_1 = 0,$$

$$\phi^3 : -\frac{6\lambda\mu b_1}{\lambda^2\sigma + \mu^2} + \frac{6\lambda\mu b_1 a_1}{\lambda^2\sigma + \mu^2} = 0,$$

$$\phi^2\psi : 12\mu a_1 - 6\mu a_1^2 + \frac{6\lambda\mu b_1^2}{\lambda^2\sigma + \mu^2} = 0,$$

$$\phi^2 : -8\lambda a_1 + 6\lambda a_1^2 + \frac{3\lambda\mu^2 a_1}{\lambda^2\sigma + \mu^2} - \frac{3\lambda\mu^2 a_1^2}{\lambda^2\sigma + \mu^2} - \frac{3\lambda^2 b_1^2}{\lambda^2\sigma + \mu^2} - a_1(4\omega + 3) = 0,$$

$$\phi\psi : -5\lambda b_1 + 6\lambda a_1 b_1 + \frac{12\lambda\mu^2 b_1}{\lambda^2\sigma + \mu^2} - \frac{12\lambda\mu^2 a_1 b_1}{\lambda^2\sigma + \mu^2} - b_1(4\omega + 3) = 0,$$

$$\phi : -\frac{6\lambda^2\mu b_1}{\lambda^2\sigma + \mu^2} + \frac{6\lambda^2\mu b_1 a_1}{\lambda^2\sigma + \mu^2} = 0,$$

$$\psi : 5\lambda\mu a_1 + 6\lambda a_1^2 - \frac{6\lambda\mu^3 a_1}{\lambda^2\sigma + \mu^2} + \frac{6\lambda\mu^3 a_1^2}{\lambda^2\sigma + \mu^2} + a_1\mu(4\omega + 3) = 0,$$

$$\phi^0 : -2\lambda^2 a_1 + 3\lambda^2 a_1^2 + \frac{3\lambda^2\mu^2 a_1}{\lambda^2\sigma + \mu^2} - \frac{3\lambda^2\mu^2 a_1^2}{\lambda^2\sigma + \mu^2} - a_1\lambda(4\omega + 3) = 0.$$

On solving the above algebraic equations using the Maple or Mathematica, we get the following results:

$$a_1 = 1, \quad b_1 = \pm\sqrt{\frac{\lambda^2\sigma + \mu^2}{-\lambda}}, \quad \omega = \frac{1}{4}(\lambda - 3).$$

In this case, the exact solution of Eq. (3.1) has the form :

$$u(\xi) = a_0 + \sqrt{-\lambda}\,\frac{A_1\cosh(\xi\sqrt{-\lambda}) + A_2\sinh(\xi\sqrt{-\lambda})}{A_1\sinh(\xi\sqrt{-\lambda}) + A_2\cosh(\xi\sqrt{-\lambda}) + \frac{\mu}{\lambda}} \pm\sqrt{\frac{\lambda^2\sigma + \mu^2}{-\lambda}}\left[\frac{1}{A_1\sinh(\xi\sqrt{-\lambda}) + A_2\cosh(\xi\sqrt{-\lambda}) + \frac{\mu}{\lambda}}\right], \qquad (3.3)$$

where a_0 is an arbitrary constant, $\sigma = A_1^2 - A_2^2$ and

$$\xi = x + y + z - \frac{1}{4}(\lambda - 3)t.$$

If $A_1 = 0$, $A_2 \neq 0$ and $\mu = 0$, then we have the solitary wave solution

$$u(\xi) = a_0 + \sqrt{-\lambda}[\tanh(\xi\sqrt{-\lambda}) \pm i\,\mathrm{sech}(\xi\sqrt{-\lambda})], \quad i = \sqrt{-1}. \qquad (3.4)$$

If $A_1 \neq 0$, $A_2 = 0$ and $\mu = 0$, then we have the solitary wave solution

$$u(\xi) = a_0 + \sqrt{-\lambda}[\coth(\xi\sqrt{-\lambda}) \pm \mathrm{cosech}(\xi\sqrt{-\lambda})]. \qquad (3.5)$$

Case 2. Trigonometric function solutions ($\lambda > 0$)

If $\lambda > 0$, substituting (3.2) into (3.1) and using (2.2) and (2.6), the left- hand side of Eq. (3.1) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to be zero, yields a system of algebraic equations in a_0, a_1, b_1, μ, λ, and ω as follows:

$$\phi^4 : -6a_1 + 3a_1^2 + \frac{3\lambda b_1^2}{\lambda^2\sigma - \mu^2} = 0,$$

$$\phi^3\psi : -6b_1 + 6a_1b_1 = 0,$$

$$\phi^3 : \frac{6\lambda\mu b_1}{\lambda^2\sigma - \mu^2} - \frac{6\lambda\mu b_1 a_1}{\lambda^2\sigma - \mu^2} = 0,$$

$$\phi^2\psi : 12\mu a_1 - 6\mu a_1^2 - \frac{6\lambda\mu b_1^2}{\lambda^2\sigma - \mu^2} = 0,$$

$$\phi^2 : -8\lambda a_1 + 6\lambda a_1^2 - \frac{3\lambda\mu^2 a_1}{\lambda^2\sigma - \mu^2} + \frac{3\lambda\mu^2 a_1^2}{\lambda^2\sigma - \mu^2} + \frac{3\lambda^2 b_1^2}{\lambda^2\sigma - \mu^2} - a_1(4\omega + 3) = 0,$$

$$\phi\psi: -5\lambda b_1 + 6\lambda a_1 b_1 - \frac{12\lambda\mu^2 b_1}{\lambda^2\sigma-\mu^2} + \frac{12\lambda\mu^2 a_1 b_1}{\lambda^2\sigma-\mu^2} - b_1(4\omega+3)=0,$$

$$\phi: \frac{6\lambda^2\mu b_1}{\lambda^2\sigma-\mu^2} - \frac{6\lambda^2\mu b_1 a_1}{\lambda^2\sigma-\mu^2}=0,$$

$$\psi: 5\lambda\mu a_1 - 6\lambda\mu a_1^2 + \frac{6\lambda\mu^3 a_1}{\lambda^2\sigma-\mu^2} - \frac{6\lambda\mu^3 a_1^2}{\lambda^2\sigma-\mu^2} + a_1\mu(4\omega+3)=0,$$

$$\phi^0: -2\lambda^2 a_1 + 3\lambda^2 a_1^2 - \frac{3\lambda^2\mu^2 a_1}{\lambda^2\sigma-\mu^2} + \frac{3\lambda^2\mu^2 a_1^2}{\lambda^2\sigma-\mu^2} - a_1\lambda(4\omega+3)=0.$$

On solving the above algebraic equations using the Maple or Mathematica, we get the following results:

$$a_1=1,\quad b_1=\pm\sqrt{\frac{\lambda^2\sigma-\mu^2}{\lambda}},\quad \omega=\frac{1}{4}(\lambda-3).$$

In this case, the exact solution of Eq. (3.1) has the form :

$$u(\xi)=a_0+\sqrt{\lambda}\,\frac{A_1\cos(\xi\sqrt{\lambda})-A_2\sin(\xi\sqrt{\lambda})}{A_1\sin(\xi\sqrt{\lambda})+A_2\cos(\xi\sqrt{\lambda})+\frac{\mu}{\lambda}} \pm\sqrt{\frac{\lambda^2\sigma-\mu^2}{\lambda}}\left[\frac{1}{A_1\sin(\xi\sqrt{\lambda})+A_2\cos(\xi\sqrt{\lambda})+\frac{\mu}{\lambda}}\right], \tag{3.6}$$

where a_0 is an arbitrary constant, $\sigma=A_1^2+A_2^2$ and

$$\xi=x+y+z-\frac{1}{4}(\lambda-3)t.$$

If $A_1=0$, $A_2\neq 0$ and $\mu=0$, then we have the solitary wave solution

$$u(\xi)=a_0-\sqrt{\lambda}[\tan(\xi\sqrt{\lambda})\pm\sec(\xi\sqrt{\lambda})]. \tag{3.7}$$

If $A_1\neq 0$, $A_2=0$ and $\mu=0$, then we have the solitary wave solution

$$u(\xi)=a_0+\sqrt{\lambda}[\cot(\xi\sqrt{\lambda})\pm co\sec(\xi\sqrt{\lambda})]. \tag{3.8}$$

Case 3. Rational function solutions □ $\lambda=0$)

If $\lambda=0$, substituting (3.2) into (3.1) and using (2.2) and (2.8), the left-hand side of Eq. (3.1) becomes a polynomial in ϕ □ and ψ. Setting the coefficients of this polynomial to the zero, yields a system of algebraic equations in a_0, a_1, b_1, μ, λ, and ω as follows:

$$\phi^4: -6a_1+3a_1^2+\frac{3b_1^2}{A_1^2-2\mu A_2}=0,$$

$$\phi^3\psi: -6b_1+6a_1b_1=0,$$

$$\phi^3: \frac{6\mu b_1}{A_1^2-2\mu A_2}-\frac{6\mu b_1 a_1}{A_1^2-2\mu A_2}=0,$$

$$\phi^2\psi: 12\mu a_1-6\mu a_1^2-\frac{6\mu b_1^2}{A_1^2-2\mu A_2}=0,$$

$$\phi^2: -\frac{3\mu^2 a_1}{A_1^2-2\mu A_2}+\frac{3\mu^2 a_1^2}{A_1^2-2\mu A_2}-a_1(4\omega+3)=0,$$

$$\phi\psi: -\frac{12\mu^2 b_1}{A_1^2-2\mu A_2}+\frac{12\mu^2 a_1 b_1}{A_1^2-2\mu A_2}-b_1(4\omega+3)=0,$$

$$\psi: \frac{6\mu^3 a_1}{A_1^2-2\mu A_2}-\frac{6\mu^3 a_1^2}{A_1^2-2\mu A_2}+a_1\mu(4\omega+3)=0.$$

On solving the above algebraic equations using the Maple or Mathematica, we get the following results:

$$a_1=1,\quad b_1=\pm\sqrt{A_1^2-2\mu A_2},\quad \omega=-\frac{3}{4}.$$

In this case, the exact solution of Eq. (3.1) has the form :

$$u(\xi)=a_0+\frac{\mu\xi+A_1\pm\sqrt{A_1^2-2\mu A_2}}{\frac{\mu}{2}\xi^2+A_1\xi+A_2}, \tag{3.9}$$

where a_0 is an arbitrary constant, and

$$\xi=x+y+z+\frac{3}{4}t.$$

Remark 5. All solutions of this paper have been checked with Maple 14 by putting them back into the original equation (1.1).

4. Conclusions

The two variable $(G'/G,1/G)$-expansion method is used in this article to obtain some new as well as some known solutions of a selected nonlinear evolution equation namely, the (3 + 1)- dimensional potential YTSF equation (1.1). As the two parameters A_1 and A_2 take special values, we obtain the solitary wave solutions. When $\mu=0$ and $b_i=0$ in Eqs. (2.1) and (2.12), the two variable $(G'/G,1/G)$-expansion method reduces to the (G'/G) expansion method. So the two variable $(G'/G,1/G)$-expansion method is an extension of the (G'/G) expansion method. The proposed method in this paper is more effective and more general than the (G'/G) -

expansion method because it gives exact solutions in more general forms. In summary, the advantage of the two variable $(G'/G,1/G)$-expansion method over the (G'/G)-expansion method is that the solutions using the first method recover the solutions using the second one.

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