Journal of Nonlinear Mathematical Physics Volume 8, Number 2 (2001), R1–R4 Book Review

Book Review by P A Clarkson

Institute of Mathematics and Statistics, University of Kent at Canterbury
United Kingdom
E-mail: P.A.Clarkson@ukc.ac.uk

Peter E Hydon: *Symmetry Methods for Differential Equations: A Beginner's Guide* Cambridge Texts in Applied Mathematics, Cambridge University Press, 2000.

In this book the author gives a straightforward introduction to symmetry methods for differential equations. Currently there are a number of other books on this subject ranging from relatively elementary texts (e.g. [13, 24]) to more substantial comprehensive works (e.g. [2, 10, 16, 22, 23]). This new book by Peter Hydon is of the former category and is emminently suitable for advanced undergraduates and beginning postgraduate students.

Although the classical Lie method of symmetry reductions is entirely algorithmic, it often involves a large amount of tedious algebra and auxiliary calculations which can become virtually unmanageable if attempted manually, and so symbolic manipulation programs have been developed, for example in *Axiom*, *Macsyma*, *Maple*, *Mathematica*, *muMATH* and *Reduce*, to facilitate the calculations; excellent reviews of the different packages available and a discussion of their strengths and applications are given in [11, 12].

Hydon's philosophy is to first consider elementary examples to illustrate an aspect of a symmetry method and then show how it can be applied to more significant examples. For example, in Chapter I, the first two ordinary differential equations considered are

$$\frac{dy}{dx} = 0, \qquad \text{and} \qquad \frac{dy}{dx} = y, \tag{1}$$

which are two of the simplest possible differential equations. Then Hydon shows how symmetry methods can be used to derive the solution of the Riccati equation

$$\frac{dy}{dx} = \frac{y+1}{x} + \frac{y^2}{x^3}, \tag{2}$$

in a very straightforward manner. Whilst the solutions of (1) are very simple, at first sight solving (2) might seem complicated though in fact its general solution is easily obtained using symmetry methods. I feel that this approach, based upon examples rather than technical details, is ideal for introducing students to symmetry methods for differential equations.

In Chapter 1, an introduction to symmetries is given, starting from symmetries of triangles and circles. However introducing the reader to the concept of a diffeomorphism on page 2 seems a little out of place in between discussing symmetries of the triangle and the circle. Lie symmetries of first-order ordinary differential equations are discussed in Chapter 2 and Lie point symmetries of higher-order ordinary differential equations are discussed in Chapter 3. In Chapter 4, Hydon discusses how to use a one-parameter Lie group including reduction of order using canonical coordinates, variational symmetries and invariant solutions. Chapters 5 and 6 are concerned with Lie symmetries involving several parameters and the solution of ordinary differential equations with multi-parameter Lie groups. In Chapter 7, symmetry techniques based on first integrals are discussed. Chapters 8 and 9 are concerned with Lie point symmetries of partial differential equations and methods for obtaining exact solutions of partial differential equations. In Chapter 10, Hydon discusses the classification of invariant solutions for both ordinary differential equations and partial differential equations. Finally in Chapter 11, discrete symmetries of differential equations are discussed. These discrete symmetries are a relatively new concept which Hydon himself has been at the forefront of developing. At the end of every chapter, there are suggestions for "Further Reading", which I liked very much, and exercises. Solutions and hints for some of the exercises are given at the end of the book. However the bibliography given is rather limited and, for example, does not include any reference to the original works of Lie. Further I feel that it would have been helpful in some of the examples discussed to have given references to the origin and physical applications of the differential equations considered. Also references to the original papers in which symmetry methods were applied to these differential equations would have been appropriate, especially for those equations where some of the details were omitted.

It is stated in the Preface that the first six chapters consist of core material for a one-semester undergraduate course and I feel that this is about right. The material in these first six chapters has clearly been well worked through and contains numerous interesting examples. However I feel that Chapters 8 and 9 on symmetry methods for partial differential equations are not quite as clear and instructive as the earlier chapters on symmetry methods for ordinary differential equations. Hydon recommends in the Preface that students should learn how to use a computer algebra package, which I feel is excellent advice. However Hydon then proceeds and only makes some rather limited comments on the use of computer algebra which I feel is a little disappointing.

Overall I thoroughly recommend this book and believe that it will be a useful textbook for introducing students to symmetry methods for differential equations. The topic is thought by some to rely on "inspired guesswork" rather than the simple algorithms which they actually are. I fully agree with Hydon's comment in the Preface that "no one who works with differential equations can afford to be ignorant of these [symmetry] methods".

References

[1] Bluman B W and Cole J D, The General Similarity Solution of the Heat Equation, *J. Math. Mech.* **18** (1969), 1025–1042.

[2] Bluman G W and Kumei S, Symmetries and Differential Equations, Springer–Verlag, New York, 1989

[3] Champagne B, Hereman W and Winternitz P, The Computer Calculation of Lie Point Symmetries of Large Systems of Differential Equations, *Comp. Phys. Comm.* **66** (1991), 319–340.

[4] Clarkson P A, Nonclassical Symmetry Reductions of the Boussinesq Equation, *Chaos, Solitons & Fractals* **5** (1995), 2261–2301.

[5] Clarkson P A and Kruskal M D New Similarity Solutions of the Boussinesq Equation, *J. Math. Phys.* **30** (1989), 2201–2213.

[6] Clarkson P A and Mansfield E L, Symmetry Reductions and Exact Solutions of a Class of Nonlinear Heat Equations *Physica* **D70** (1993), 250–288.

[7] Clarkson P A and Mansfield E L, Algorithms for the Nonclassical Method of Symmetry Reduction, *SIAM J. Appl. Math.* **54** (1994), 1693–1719.

[8] Clarkson P A and Winternitz P, Symmetry Reduction and Exact Solutions of Nonlinear Partial Differential Equations, in The Painlevé Property, One Century LaterEditor R Conte *CRM Series in Mathematical Physics*, Springer-Verlag, New York, 1999, 591–660

[9] Dresner L, Applications of Lie's Theory of Ordinary and Partial Differential Equations, Institute of Physics, Bristol, 1999.

[10] Fushchich W I, Shtelen W M and Serov N I, Symmetry Analysis and Exact Solutions of the Equations of Mathematical Physics, Kluwer, Dordrecht, 1993.

[11] Hereman W, Review of Symbolic Software for the Computation of Lie Symmetries of Differential Equations, *Euromath Bull.* **1** (1994), 45–79.

[12] Hereman W, Symbolic Software for Lie Symmetry Analysis, in Lie Group Analysis of Differential equations. III. New Trends in Theoretical Developments and Computational MethodsEditor Ibragimov N H, CRC Press, Boca Raton, Chapter XII, 1996, 367–413.

[13] Hill J M, Differential Equations and Group Methods for Scientists and Engineers, CRC Press, Boca Raton, 1992.

[14] Hydon P E, Symmetry Methods for Differential Equations *Texts in Applied Math.*, C.U.P., Cambridge, 2000

[15] Ibragimov N H, Elementary Lie Group Analysis and Ordinary Differential Equations, Wiley, Chichester, 1999.

[16] Ibragimov N H (Editor), CRC Handbook of Lie Group Analysis of Differential Equations. Volumes I, II, III, CRC Press, Boca Raton, Vol.I 1994, Vol.II 1995, Vol.III 1996.

[17] Janet M, *J. Math. Pure Appl.* **3** (1920), 65–151.

[18] Levi D and Winternitz P, Nonclassical Symmetry Reduction: Example of the Boussinesq Equation, *J. Phys. A: Math. Gen.* **22** (1989), 2915–2924.

[19] Mansfield E L, The Differential Algebra Package *diffgrob2*, *MapleTech* **3** (1996), 33-37.

[20] Mansfield E L, The Nonclassical Group Analysis of the Heat Equation, *J. Math. Anal. Appl.* **231** (1999), 526–542.

[21] Mansfield E L and Clarkson P A, Applications of the Differential Algebra Package *diffgrob2* to Classical Symmetries of Differential Equations, *J. Symb. Comp.* **23** (1997), 517–533.

[22] Olver P J, Application of Lie Groups to Differential Equations, Second Edition, Springer–Verlag, New York, 1993.

[23] Ovsiannikov L V, Group Analysis of Differential Equations [Translated by Ames W F], Academic, New York, 1982.

[24] Stephani H, Differential Equations, Their Solution using Symmetries[Ed. MacCallum M], C.U.P., Cambridge, 1989.

Peter Clarkson, February 2001.