A New Model of Isoseismal Area Assessment Based on Information Granule Diffusion*

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Abstract

Utilizing the technique of information granule diffusion and fuzzy inference with max-min operation, this paper provides a new model to estimate isoseismal area by earthquake magnitude. The model does not depend on any extra condition but scanty historical earthquake recorders. The technique of information granule diffusion is employed to fill up the gaps which a small sample caused. The fuzzy inference can avoid complex computation. In this paper, we also demonstrate the benefit of the suggested model by comparing their results of linear regression and hybrid fuzzy-neural-network method.

Keywords: Information granule, Fuzzy inference, Isoseismal area

1 Introduction

Earthquake has uncertainty, unexpectedness, tremendous damage and widely social influence, and also earthquake prediction is very hard. Therefore, it is one of the most dangerously natural disasters which threaten man's existence and development. Isoseismal area is an important parameter which weighes the scope of affected region. It can response both the size of earthquake energy and the attenuation change of intensity.

During the past years, accompanied by the intensity attenuation analysis, the relationship between isoseismal area and earthquake magnitude has been a major area of concern which requires thorough investigation. Early in 1970s, engineers [1] in earthquake engineering expressed the relationship between intensity, I, and magnitude, M, and hypocentral distance, R (in kilometers), by the following formula

$$I = aM - b\log_{10} R + c$$

where a, b, and c are empirical constants. Since there is a 60% probability that an observed intensity is bigger or smaller than its predicted value [2], a more appropriate expression [3] relating isoseismal area, S (in square kilometers), with intensity, I, to magnitude, M, was developed

$$\log_{10} S(I) = a + bM$$

where a and b are empirical constants.

However, some studies have demonstrated that the relationship between isoseismal area and magnitude is not simple linear relationship. Some researchers established regression analysis models based on powerful statistical tools in order to reveal the nonlinear relationship. These models must have quantities of observations, or not, their results are insignificant at all. In fact, destructive earthquakes are infrequent events with very small probability of occurrence. In other words, observations used in estimating isoseismal area are incomplete, and form a small sample.

To resolve this problem, fuzzy models have been formulated to study the uncertainty and ambiguity of isoseismal maps [4]. However, different researchers usually obtain different results based on the same observations because their analyses depend on personal engineering experience rather than observations.

In view of this point, Huang and Leung [5] provided a hybrid fuzzy-neural-network method which is very effective in estimating the relationship between isoseismal area and earthquake magnitude although data are usually scanty, incomplete, and contradictory.

But, when a trained neural network is performing as a mapping from input space to output space, it is a black box. This means it is not possible to understand how a neural system works, and it is very hard to incorporate human a prior knowledge into this method. Furthermore, granularity of the observations is not considered.

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In this paper, we propose a new model to these problems as a whole. In section 2, fuzzy inference model based on information granule diffusion method is provided in detail. An application in estimating isoseismal area by earthquake magnitude is stated in section 3. In section 4, we compare theirs results of linear regression and hybrid fuzzy-neural-network method in order to demonstrate the benefits of the new model. The paper is then summarized with a conclusion in section 5.

2 Fuzzy Inference Model Based on Information Granule Diffusion

Information granule diffusion method [6] can turn a crisp information granule into a fuzzy information granulation, and become a kind of fuzzy mathematics processing method which can come into being a set-valued sample. The simplest method is linear diffusion method .

Definition 1 Let x be an information granule with a discrete universe $U = \{u_1, u_2, \cdots, u_n\} \subset \mathbb{R}$, $\Delta \equiv u_{i+1} - u_i$, $i = 1, 2, \cdots, n-1$.

$$\mu(x, u_i) = \begin{cases} 1- \mid x - u_i \mid /\Delta, & \text{if } |x - u_i| \le \Delta; \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

is called linear diffusion of information granule x on U. $\mu(x, u)$ is called a diffusion function.

In the following section, we detailedly describe fuzzy inference model based on information granule diffusion.

Firstly, in estimating isoseismal area by magnitude, m, in order to reduce scattering in the sample, we generally consider logarithmic area, $y = \log_{10} S$, instead of S. What's more, isoseismal area doesn't exist when epicentral intensity, I, is less than VI according to recording historical earthquakes. Therefore, isoseismal area is estimated only for $I \geq$VI, $I \geq$VII, $I \geq$VIII or $I \geq$IX. Secondly, for convenience of expression, magnitude granule and logarithmic area granule are still denoted m and y.

Then, let the sample with magnitude $M = (m_1, m_2, \cdots, m_n)$ and logarithmic isoseismal area $Y = (y_1, y_2, \cdots, y_n)$ be in Eq.(2).

$$W = \{(m_1, y_1), (m_2, y_2), \cdots, (m_n, y_n)\} \tag{2}$$

where n is the number of earthquake records.

For M, let a discrete universe be in Eq.(3).

$$U = \{u_1, u_2, \cdots, u_{l_1}\} \subset \mathbb{R} \tag{3}$$

where $\Delta_m \equiv u_{i+1} - u_i, i = 1, 2, \cdots, l_1 - 1$.

For Y, let a discrete universe be in Eq.(4).

$$V = \{v_1, v_2, \cdots, v_{l_2}\} \subset \mathbb{R} \tag{4}$$

where $\Delta_y \equiv v_{j+1} - v_j, j = 1, 2, \cdots, l_2 - 1$.

According to Eq.(1), we can obtain diffusion functions of information granule m_k and y_k, $k = 1, 2, \cdots, n$, in Eq.(5) and Eq.(6), respectively.

$$\mu_{m_k}(m_k, u_i) = \begin{cases} 1- \mid m_k - u_i \mid /\Delta_m, \\ \quad \text{if } |m_k - u_i| \le \Delta_m; \\ 0, \quad \text{otherwise,} \end{cases} \tag{5}$$

$$\mu_{y_k}(y_k, v_j) = \begin{cases} 1- \mid y_k - v_j \mid /\Delta_y, \\ \quad \text{if } |y_k - v_j| \le \Delta_y; \\ 0, \quad \text{otherwise,} \end{cases} \tag{6}$$

Therefore, for information granule (m_k, y_k), we suppose diffusion function in Eq.(7).

$$\mu_{Q_k}(u_i, v_j) = \mu_{m_k}(u_i) \times \mu_{y_k}(v_j) \tag{7}$$

Hence,

$$Q_k = (q_{kij})_{(l_1-1)\times(l_2-1)} \tag{8}$$

Let

$$q_{ij} = \sum_{k=1}^{n} q_{kij} \tag{9}$$

To normalize, we show in eq.(10)

$$Q = Q_{ij} = q_{ij} / \max_{1 \le i \le l_1 - 1} q_{ij} \tag{10}$$

When magnitude m_0 is given, we can turn m_0 into a fuzzy information granulation $\widetilde{m}_0$ by using the formula in eq.(1).

Employing max-min composition shown in Eq.(11), we obtain a fuzzy consequent $\widetilde{y}_0$ based on Q.

$$\widetilde{y}_0 = \max_{u_i \in U}\{\min\{\mu_{\widetilde{m}_0}(u_i), Q_{ij}\}\} \tag{11}$$

Finally, we calculate the gravity center y_0 of $\widetilde{y}_0$, which is

$$y_0 = \Big(\sum_{j=1}^{l_2-1} \mu_{\widetilde{y}_0}(v_j) \times v_j\Big) / \Big(\sum_{j=1}^{l_2-1} \mu_{\widetilde{y}_0}(v_j)\Big). \tag{12}$$

It means that estimate value of logarithmic isoseismal area is equal to y_0 for magnitude m_0. In other words, it is y_0 that we have got the estimate value by fuzzy inference based on diffusion method of magnitude granule m_0. we call it to be information granule diffusion fuzzy inference (IGDFI) model which comprises Eqs.(3-12).

3 Estimating Isoseismal Area

In Yunnan Province of China, there is a data set of strong earthquakes consisting of 25 records from 1913 to 1976 with magnitude, M, and isoseismal area, S, of intensity, $I \geq VII$ (Table 1).

Table 1: Magnitudes and isoseismal areas

No.	M	$S_{I\geq VII}$	No.	M	$S_{I\geq VII}$
1	6.5	2,848	14	6.1	733
2	6.5	3,506	15	5.1	19
3	7	4,758	16	6.5	1,703
4	5.75	779	17	5.4	261
5	7	2,593	18	6.4	404
6	7	1,656	19	7.7	8,176
7	6.25	3,385	20	5.5	100
8	6.25	1,345	21	6.7	212
9	5.75	190	22	5.5	18
10	6	88	23	6.8	200
11	5.8	47	24	7.1	837
12	6	3,582	25	5.7	99
13	6.2	449			

M is the Richter magnitude scale,
S is square kilometers.

According to Table 1, we obtain the sample W in Eq.(13) utilizing the formula (2).

$$\begin{aligned} W &= \{(m_1,y_1),(m_2,y_2),\cdots,(m_{25},y_{25})\} \\ &= \{(6.5,3.455),(6.5,3.545),(7,3.677),(5.75,\\ &\quad 2.892),(7,3.414),(7,3.219),(5.75,2.279),\\ &\quad (6,1.944),(5.8,1.672),(6,3.554),(6.25,\\ &\quad 3.530),(6.25,3.129),(6.2,2.652),(6.1,2.865),\\ &\quad (5.1,1.279),(6.5,3.231),(5.4,2.417),(6.4,\\ &\quad 2.606),(7.7,3.913),(5.5,2.000),(6.7,2.326),\\ &\quad (5.5,1.255),(6.8,2.301),(7.1,2.923),\\ &\quad (5.7,1.996)\}. \end{aligned} \tag{13}$$

In order to be compared with hybrid fuzzy-neural-network method, we select discrete universes U and V in Eq.(14) and Eq.(15) for diffusing information granule (m_k, y_k), $k = 1, 2, \cdots, 25$.

$$U = \{u_1, u_2, \cdots, u_{30}\} = \{5.010, 5.106, \cdots, 7.758\} \tag{14}$$

where step length Δ_m is 0.096.

$$V = \{v_1, v_2, \cdots, v_{30}\} = \{1.164, 1.262, \cdots, 4.006\} \tag{15}$$

where step length Δ_y is 0.098.

By Eqs.(5) and (6), we obtain diffusion functions of m_k and y_k in Eqs.(16) and (17).

$$\mu_{m_k}(m_k, u_i) = \begin{cases} 1-\mid m_k - u_i \mid /0.096, \\ \qquad \text{if } |m_k - v_i| \leq 0.096; \\ 0, \qquad \text{otherwise,} \end{cases} \tag{16}$$

$$\mu_{y_k}(y_k, v_j) = \begin{cases} 1-\mid y_k - v_j \mid /0.098, \\ \qquad \text{if } |y_k - v_j| \leq 0.098; \\ 0, \qquad \text{otherwise,} \end{cases} \tag{17}$$

For example, suppose $k = 1$, we have Q_1 in Eq.(18) by employing Eqs.(16), (17) and (7).

$$Q_1 = \begin{array}{c} \\ u_{16} \\ u_{17} \end{array} \begin{array}{c} \begin{array}{cc} v_{24} & v_{25} \end{array} \\ \begin{pmatrix} 0.0755 & 0.0287 \\ 0.6490 & 0.2468 \end{pmatrix} \end{array} \tag{18}$$

It is noted that the other elements of Q_1 that are not shown in Eq.(18) are equal to 0. Thus, we can obtain $Q_2, Q_3, \cdots, Q_{25}$ according to different k. By formulas (9) and (10), summing and normalizing and finally expressing in Eq.(19).

$$Q = \begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_{30} \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & \cdots & v_{30} \end{array} \\ \begin{pmatrix} 0.000000 & 0.062101 & \cdots & 0.000000 \\ 0.000000 & 0.931510 & \cdots & 0.000000 \\ \vdots & \vdots & \vdots & \vdots \\ 0.000000 & 0.000000 & \cdots & 0.655183 \end{pmatrix} \end{array} \tag{19}$$

If $m_0 = 6.5$, then

$$\begin{aligned} \widetilde{m}_0 &= 0/5.010 + \cdots + 0/6.354 + 0.7218/6.414 \\ &+0.2782/6.510 + 0/6.616 + \cdots + 0/7.758 \end{aligned} \tag{20}$$

By formula (11),

$$\begin{aligned} \widetilde{y}_0 &= 0/1.164 + \cdots + 0.8958/3.32 + 0.648936/3.418 \\ &+0.8958/3.516 + 0.828571/3.614 + \cdots + 0/4.006 \end{aligned} \tag{22}$$

Finally, we compute approximate value y_0 with Eq.(12).

$$\begin{aligned} y_0 &= (0\times 1.164 + \cdots + 0.8958\times 3.32 + 0.648936\times 3.418 \\ &+0.8958 \times 3.516 + 0.828571 \times 3.614 + \cdots + 0\times \\ &4.006)/(0 + \cdots + 0.8958 + 0.648936 + 0.8958 \\ &+0.828571 + \cdots + 0) = 3.371745 \end{aligned}$$

It means that, in Yunnan province of China, according to historical earthquake experience, if an earthquake of Richter magnitude scale $m = 6.5$ occurs, the isoseismal area of intensity I≥VII caused by this earthquake is about $S = 10^{3.371745} = 2353$ km^2.

4 Compared with other models

In this section, in order to show the superiority of the model, we have the three results of our model, linear regression and hybrid fuzzy-neural-network for the sample W in Eq.(13). For comparison, Table 2 is described and the average of squared errors are calculated.

Firstly, for all magnitude in Table 1, we apply our model to estimate isoseismal area, and show its results Y_{IGDFI} in the fifth volume of Table 2.

Secondly, the estimate values Y_{LR} of linear regression which is $y = -2.61 + 0.85m$ are displayed in the third volume of Table 2.

Finally, the computational results Y_{HM} of hybrid fuzzy-neural-network method which sets the momentum rate $\eta = 0.9$ and learning rate $\alpha = 0.7$ are shown in the fourth volume of Table 2.

Table 2: True values and estimated values of isoseismal areas

M	Y	Y_{LR}	Y_{HM}	Y_{IGDFI}
6.5	3.455	2.915	3.108565	3.371745
6.5	3.545	2.915	3.108565	3.371745
7	3.677	3.34	3.170555	3.456446
5.75	2.892	2.2775	2.217484	2.142923
7	3.414	3.34	3.170555	3.456446
7	3.219	3.34	3.170555	3.456446
6.25	3.53	2.7025	3.016197	3.12114
6.25	3.129	2.7025	3.016197	3.12114
5.75	2.279	2.2775	2.217484	2.142923
6	1.944	2.49	2.673021	2.596824
5.8	1.672	2.32	2.296665	2.151083
6	3.554	2.49	2.673021	2.596824
6.2	2.652	2.66	2.967548	2.964908
6.1	2.865	2.575	2.840106	2.696019
5.1	1.279	1.725	1.447158	1.312737
6.5	3.231	2.915	3.108565	3.371745
5.4	2.417	1.98	1.903090	2.340655
6.4	2.606	2.83	3.111934	2.754144
7.7	3.913	3.935	3.895533	3.957
5.5	2	2.065	1.968483	1.601353
6.7	2.326	3.085	2.851870	2.242
5.5	1.255	2.065	1.968483	1.601353
6.8	2.301	3.17	2.884229	2.242
7.1	2.923	3.425	3.251395	3.09582
5.7	1.996	2.235	2.149219	2.13942
E		0.273673	0.199351	0.11918

M is earthquake magnitude,
Y is logarithmic isoseismal area,
Y_{LR} is the result of linear regression model,
Y_{HM} is the result of hybrid fuzzy-neural-network model,
Y_{IGDFI} is the result of information granule diffusion fuzzy inference model,
E is mean square error.

In order to contrast, the mean square errors E of the three estimators: the linear-regression estimator(LR); the hybrid fuzzy-neural-network estimator(HM) and the information granule diffusion fuzzy inference estimator(IGDFI) are computed as follows:

$$\begin{cases} E_{LR} = \frac{1}{25}\sum_{i=1}^{25}(y_i - y_{LRi})^2 = 0.273673, \\ E_{HM} = \frac{1}{25}\sum_{i=1}^{25}(y_i - y_{HMi})^2 = 0.199351, \\ E_{IGDFI} = \frac{1}{25}\sum_{i=1}^{25}(y_i - y_{IGDFIi})^2 = 0.11918. \end{cases}$$

Obviously, the IGDFI estimator is better than the linear-regression estimator and the hybrid fuzzy-neural-network estimator, because the IGDFI estimator is more precise, nearer to real value, and have the following benefits: 1) Avoiding complex computation. Thus, it is easy to be applied for any other researchers. 2) Applying information granule diffusion method. Thus, fuzziness and granularity of the observations have been resolved. 3) Without any extra condition except for the historical observations. 4) Being more visual.

5 Conclusion and Discussion

In this paper, we have sated an IGDFI model to estimate isoseismal area with scanty, incomplete and inconsistent earthquake recordings. The basic advantage of using information granule is that an observation is looked on as an information granule which naturally fills up the information gaps caused by inconsistent data. Information granule diffusion method turns a crisp information granule into a fuzzy information granulation which further resolves fuzziness and incompleteness of observations. Fuzzy inference approach employs max-min operation which avoids complex computation.

Through an application in estimating isoseismal area by magnitude in Yunnan province of China, it has been demonstrated that the result of IGDFI model is nearer to real value than their results of linear regression and hybrid fuzzy-neural-network for scanty, incomplete and inconsistent data. What's more, the computational process of IGDFI model is more visual than the one of hybrid fuzzy-neural-network model. The model can be also applicable to other small sample problems.

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