$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!n!} E\{\{S_T \exp\{-\mu T\} - K\exp\{-rT\}\} I_{\{\xi<d^{(m)},\gamma>c^{(m,n)}\}} \big| Q_{T_1}=m, Q_{T-T_1}=n\}$$
$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!n!}\{V_{31}-V_{32}\},$$

where

$$V_{31} = E\{S_T\exp\{-\mu T\} I_{\{\xi<d^{(m)},\gamma>c^{(m,n)}\}} \big| Q_{T_1}=m, Q_{T-T_1}=n\} = E\{S_0\exp\{-\frac{1}{2}(\sigma^{(m,n)})^2+\sigma^{(m,n)}\gamma\} I_{\{\xi<d^{(m)},\gamma>c^{(m,n)}\}}\}$$
$$= S_0\exp\{-\frac{1}{2}(\sigma^{(m,n)})^2\}\int_{-\infty}^{d^{(m)}}\int_{c^{(m,n)}}^{+\infty}\exp\{\sigma^{(m,n)}v\}\varphi(u,v,\rho_2^{(m,n)})dudv = S_0\int_{-\infty}^{d^{(m)}}\int_{c^{(m,n)}}^{+\infty}\varphi(u-\rho_2^{(m,n)}\sigma^{(m,n)}, v-\sigma^{(m,n)},\rho_2^{(m,n)})dudv$$
$$= S_0\int_{-\infty}^{d^{(m)}-\rho_2^{(m,n)}\sigma^{(m,n)}}\int_{c^{(m,n)}-\sigma^{(m,n)}}^{+\infty}\varphi(u,v,\rho_2^{(m,n)})dudv = S_0\Phi(d^{(m)}-\rho_2^{(m,n)}\sigma^{(m,n)}, \sigma^{(m,n)}-c^{(m,n)}, -\rho_2^{(m,n)}),$$
$$V_{32} = E\{K\exp\{-rT\} I_{\{\xi<d^{(m)},\gamma>c^{(m,n)}\}} \big| Q_{T_1}=m, Q_{T-T_1}=n\} = K\exp\{-rT\}P\{\xi<d^{(m)},\gamma>c^{(m,n)}\} = K\exp\{-rT\}\Phi(d^{(m)}, -c^{(m,n)}, -\rho_2^{(m,n)}).$$

Remark 7 When the jump intensity $\lambda=0$, we get the actuarial price of reload option in the fractional Brownian motion environment

$$V_0 = S_0\Phi(-d+\sigma_1) - K\exp\{-rT_1\}\Phi(-d) + K\Phi(\rho_1\sigma_2-d, \sigma_2-a, \rho_1) - K\exp\{-rT\}\Phi(-d,-a,\rho_1)$$
$$+ S_0\Phi(d-\rho_2\sigma_3, \sigma_3-c, -\rho_2) - K\exp\{-rT\}\Phi(d,-c,-\rho_2),$$

where

$$d = \{\ln(K/S_0) - rT_1 + \frac{1}{2}\sigma^2T_1^{2H}\}\Big/\sigma_1, \quad a = \{-rT+\frac{1}{2}\sigma^2(T^{2H}-T_1^{2H})\}\Big/\sigma_2, \quad c = \{\ln(K/S_0) - rT_1 + \frac{1}{2}\sigma^2T_1^{2H}\}\Big/\sigma_3,$$
$$\sigma_1 = \sigma T_1^H, \quad \sigma_2 = \sigma\sqrt{(T^{2H}-T_1^{2H})}, \quad \sigma_3 = \sigma T^H,$$
$$\rho_1 = \{T^{2H}-T_1^{2H}-(T-T_1)^{2H}\}\big/\{2T_1^H\sqrt{(T^{2H}-T_1^{2H})}\}, \quad \rho_2 = \{T^{2H}+T_1^{2H}-(T-T_1)^{2H}\}\big/\{2T_1^HT^H\}.$$

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References

[1] Feng G, Liu Z, and Hou Z. Reload stock options pricing with underlying stock asset obeying jump-diffusion process [J]. Journal of Systemms Engineering, 2003, 18(1) 91-93.

[2] Wang X, Du X. Pricing the reload options under jump-diffusion model [J]. Mathematics in Economics, 2007, 24(3) 276-282.

[3] Bladt M, Rydberg T. An actuarial approach to option pricing under the physical measure and without market assumptions [J]. Insurance: Mathematics and Economics, 1998, 22(1) 65-73.

[4] Jiao L, Liang X, Jiang L. Improved reload option pricing under jump-diffusion model [J]. Journal of Shandong University of Technology, 2009, 23(6) 86-89.

[5] Fang Z, He C, Wang Y. Compound option pricing on fractional jump-diffusions [J]. Mathematics in Practice and Theory, 2011, 41(12) 6-14.

[6] Xue H, Lu J, Li Q, Wang X. Fractional Jump-diffusion Pricing Model under Stochastic Interest Rate [R]. Prceedings 2011 3rd International Conference on Information and Financial Engineering, Shanghai, August, 2011, 428-432.

[7] Fu Q, Shi Z. An actuarial approach fractional geometric Asian-reload stock option and it’s simulation [J]. Journal of Chongqing University, 2010, 16(6) 22-25.