

A New Method for Ranking Generalized Fuzzy Numbers for Handling Fuzzy Risk Analysis Problems

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Abstract

In this paper, we present a new method for ranking generalized fuzzy numbers for dealing with fuzzy risk analysis problems. The proposed method considers the defuzzified values, the heights and the spreads of generalized fuzzy numbers, simultaneously, for ranking generalized fuzzy numbers. It gets better ranking results to rank generalized fuzzy numbers than the existing methods. We also apply the proposed method to deal with fuzzy risk analysis problems. The proposed method provides a useful way for handling fuzzy risk analysis problems.

Keywords: Defuzzified Values, Fuzzy Risk Analysis, Generalized Fuzzy Numbers, Ranking Order, Standardized Generalized Fuzzy Numbers.

1. Introduction

To deal with fuzzy risk analysis problems, experts generally use fuzzy numbers to represent the evaluating value of the risk of each subcomponent made by each manufactory. In order to properly evaluate the risk of components made by manufactories, many methods have been presented for ranking fuzzy numbers [1]-[4], [7]-[15]. Although many methods for ranking fuzzy numbers have been presented, in [3] and [10], Chen et al. pointed out that there are still some drawbacks for the existing methods.

In this paper, in order to overcome the drawbacks of the existing methods, we propose a new method for ranking generalized fuzzy numbers. The proposed method considers the defuzzified values, the heights and the spreads of generalized fuzzy numbers, simultaneously, to rank generalized fuzzy numbers. The experts can set the degree of importance α of the spreads of the fuzzy numbers for ranking fuzzy numbers, where $\alpha = \{0.5, 1.5\}$. If the experts consider that the spread is more importance, then they can set the value of $\alpha = 1.5$. Otherwise, they can set the value of $\alpha = 0.5$. The proposed method gets better ranking

results to rank generalized fuzzy numbers than the existing methods. We also apply the proposed method to deal with fuzzy risk analysis problems. The proposed method provides a useful way for handling fuzzy risk analysis problems.

2. A New Method for Ranking Generalized Fuzzy Numbers

In 1985, Chen [2] proposed the concept of generalized fuzzy numbers. Let \tilde{A} be a generalized trapezoidal fuzzy number, $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$, as shown in Fig. 1, where a_1, a_2, a_3, a_4 are real values, $w_{\tilde{A}}$ denotes the height of the generalized trapezoidal fuzzy number \tilde{A} , and $w_{\tilde{A}} \in [0, 1]$. If $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4$, then \tilde{A} is called a standardized generalized fuzzy number. If $w_{\tilde{A}} = 1$, then \tilde{A} is called a normalized fuzzy number and it can be represented as $\tilde{A} = (a_1, a_2, a_3, a_4)$. If $a_2 = a_3$, then \tilde{A} is a triangular fuzzy number. If $a_1 = a_2 = a_3 = a_4$, then \tilde{A} is a crisp value.

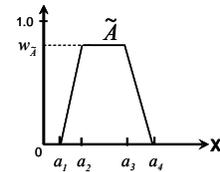


Fig. 1: A generalized trapezoidal fuzzy number \tilde{A}

If \tilde{A} is a trapezoidal fuzzy number, i.e., $\tilde{A} = (a_1, a_2, a_3, a_4)$, then the defuzzified value $x_{\tilde{A}}$ of the fuzzy number \tilde{A} can be calculated as follows [6]:

$$x_{\tilde{A}} = \frac{a_1 + a_2 + a_3 + a_4}{4}. \quad (1)$$

In this section, we present a new method for ranking generalized fuzzy numbers, where we consider three factors for ranking generalized fuzzy numbers, simultaneously, i.e., the defuzzified values of generalized fuzzy numbers, the height of generalized fuzzy numbers, and the spreads of generalized fuzzy numbers. Assume that there are n generalized trapezoidal fuzzy numbers A_1, A_2, \dots , and A_n to be ranked, where $A = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ and $1 \leq i \leq$

n . The proposed method for ranking generalized trapezoidal fuzzy numbers is now presented as follows:

Step 1: Standardize each generalized fuzzy number \tilde{A}_i into standardized generalized fuzzy numbers \tilde{A}_i^* , shown as follows:

$$\begin{aligned}\tilde{A}_i^* &= \left(\frac{a_{i1}}{k}, \frac{a_{i2}}{k}, \frac{a_{i3}}{k}, \frac{a_{i4}}{k}; w_{\tilde{A}_i} \right) \\ &= (a_{i1}^*, a_{i2}^*, a_{i3}^*, a_{i4}^*; w_{\tilde{A}_i^*}),\end{aligned}\quad (2)$$

where k denotes the maximum value of the universe of discourse and $1 \leq i \leq n$.

Step 2: Calculate the defuzzified value $x_{\tilde{A}_i^*}$ of each standardized generalized fuzzy number \tilde{A}_i^* , shown as follows:

$$x_{\tilde{A}_i^*} = \frac{a_{i1}^* + a_{i2}^* + a_{i3}^* + a_{i4}^*}{4}, \quad (3)$$

where $1 \leq i \leq n$.

Step 3: Calculate the spread $STD_{\tilde{A}_i^*}$ of each standardized generalized fuzzy number \tilde{A}_i^* , shown as follows:

$$STD_{\tilde{A}_i^*} = \sqrt{\frac{\sum_{j=1}^4 (a_{ij}^* - x_{\tilde{A}_i^*})^2}{4-1}}, \quad (4)$$

where $1 \leq i \leq n$.

Step 4: Calculate the $Score(\tilde{A}_i^*)$ of each standardized generalized fuzzy number \tilde{A}_i^* , shown as follows:

$$Score(\tilde{A}_i^*) = x_{\tilde{A}_i^*} \times w_{\tilde{A}_i^*} \times (1 - \alpha \cdot STD_{\tilde{A}_i^*}), \quad (5)$$

where α is a parameter for adjusting the degree of importance of the spreads of the generalized fuzzy numbers and $\alpha \in \{0.5, 1.5\}$. If the experts consider that the spread is a more important factor, then they can set the value of $\alpha = 1.5$. Otherwise, they can set the value of $\alpha = 0.5$. The larger the value of $Score(\tilde{A}_i^*)$, the better the ranking of \tilde{A}_i^* , where $1 \leq i \leq n$.

In the following, we use an example to illustrate the ranking process of the proposed method. Assume that there are two generalized fuzzy numbers $\tilde{A} = (1, 3, 4, 5; 0.6)$ and $\tilde{B} = (2, 3, 4, 5; 0.8)$ as shown in Fig. 2(a). Because the maximum value of the universe of discourse X is 5 (i.e., $k = 5$), based on Eq. (2), the generalized fuzzy numbers of \tilde{A} and \tilde{B} are transformed into standardized generalized fuzzy numbers \tilde{A}^* and \tilde{B}^* , respectively, where $\tilde{A}^* = (0.2, 0.6, 0.8, 1.0; 0.6)$ and $\tilde{B}^* = (0.4, 0.6, 0.8, 1.0; 0.8)$ as shown in Fig. 2(b). Then, based on Eq. (3), we can calculate the centers of gravity $x_{\tilde{A}^*}$ and $x_{\tilde{B}^*}$ of \tilde{A}^* and \tilde{B}^* , respectively, where $x_{\tilde{A}^*} = 0.65$ and $x_{\tilde{B}^*} = 0.70$.

Then, based on Eq. (4), we can calculate the spread $STD_{\tilde{A}^*}$ and $STD_{\tilde{B}^*}$ of \tilde{A}^* and \tilde{B}^* , respectively, where $STD_{\tilde{A}^*} = 0.3416$ and $STD_{\tilde{B}^*} = 0.2582$. Finally, based on

Eq. (5), we can get the scores $Score(\tilde{A}^*)$ and $Score(\tilde{B}^*)$ of \tilde{A}^* and \tilde{B}^* , respectively, where

$$Score(\tilde{A}^*) = 0.65 \times 0.6 \times (1 - 1.5 \times 0.3416) = 0.1902,$$

$$Score(\tilde{B}^*) = 0.7 \times 0.8 \times (1 - 1.5 \times 0.2582) = 0.3431.$$

Because $Score(\tilde{B}^*) > Score(\tilde{A}^*)$, we can see that the ranking order between the fuzzy numbers \tilde{A} and \tilde{B} is " $B > A$ ".

From this example, we can see that the larger the center of gravity, or the larger the heights, or the smaller the spreads of a fuzzy number, the higher the value of the ranking score of the fuzzy number.

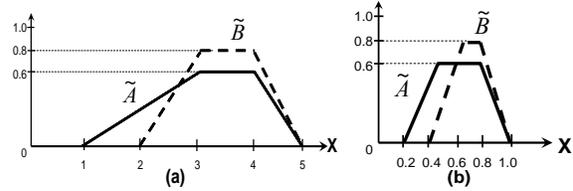


Fig. 2: Generalized fuzzy numbers. (a) Before the standardization process. (b) After the standardization process

3. Handling Fuzzy Risk Analysis Problems Based on the Proposed Fuzzy Ranking Method

In this section, we apply the proposed method for ranking generalized fuzzy numbers to deal with fuzzy risk analysis problems, where the evaluating values of fuzzy risk analysis problems are often represented by fuzzy numbers. Assume that there are n manufactories C_1, C_2, \dots , and C_n and assume that component A_i made by manufactory C_i consists of p sub-components A_{i1}, A_{i2}, \dots , and A_{ip} . The component A_i made by manufactory C_i is used to analyze the probability of failure R_{ik} of each component A_i made by manufactory C_i , where $1 \leq i \leq n$. We use two evaluating indices R_{ik} and W_{ik} to evaluate each sub-component A_{ik} , where R_{ik} and W_{ik} are generalized fuzzy numbers, R_{ik} denotes the probability of failure of the sub-component A_{ik} of each manufactory C_i , W_{ik} denotes the severity of loss of the sub-component A_{ik} of each manufactory C_i , $1 \leq k \leq p$, and $1 \leq i \leq n$. In the following, we briefly review arithmetic operations between fuzzy numbers from [2] and [5]. Assume that there are two generalized $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_{\tilde{B}})$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$, are real values, $0 \leq w_{\tilde{A}} \leq 1$, and $0 \leq w_{\tilde{B}} \leq 1$. The arithmetic operations set between \tilde{A} and \tilde{B} are as follows:

Generalized Fuzzy Numbers Addition \oplus :

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \oplus (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_{\tilde{A}}, w_{\tilde{B}})).\end{aligned}\quad (6)$$

Generalized Fuzzy Numbers Multiplication \otimes :

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \otimes (b_1, b_2, b_3, b_4; w_{\tilde{B}})$$

$$= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4; \min(w_{\tilde{A}}, w_{\tilde{B}})). \quad (7)$$

Generalized Fuzzy Numbers Division \oslash :

$$\begin{aligned} \tilde{A} \oslash \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \oslash (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1; \min(w_{\tilde{A}}, w_{\tilde{B}})). \end{aligned} \quad (8)$$

The proposed fuzzy risk analysis algorithm is now presented as follows:

Step 1: Based on the fuzzy numbers R_{ik} and W_{ik} of each sub-component A_{ik} of each component A_i made by manufactory C_i , where $1 \leq k \leq p$ and $1 \leq i \leq n$, calculate the probability of failure R_i of each component A_i made by manufactory C_i , shown as follows:

$$\tilde{R}_i = \left(\sum_{k=1}^p \tilde{R}_{ik} \otimes \tilde{W}_{ik} \right) \oslash \left(\sum_{k=1}^p \tilde{W}_{ik} \right), \quad (9)$$

where $\tilde{R}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4}; w_{\tilde{R}_i})$ and $1 \leq i \leq n$.

Step 2: Standardize each generalized fuzzy number R_i into \tilde{R}_i^* by Eq. (2), where $1 \leq i \leq n$.

Step 3: Calculate the defuzzified value $(x_{\tilde{R}_i^*}, y_{\tilde{R}_i^*})$ of each standardized generalized fuzzy number \tilde{R}_i^* by Eq. (3), where $1 \leq i \leq n$.

Step 4: Calculate the spreads $STD_{\tilde{R}_i^*}$ of each standardized generalized fuzzy number \tilde{R}_i^* by Eq. (4), where $1 \leq i \leq n$.

Step 5: Calculate the $Score(\tilde{R}_i^*)$ of each standardized generalized fuzzy number \tilde{R}_i^* by Eq. (5), where $1 \leq i \leq n$. The larger the value $Score(\tilde{R}_i^*)$, the higher the risk of component A_i made by manufactory C_i , where $1 \leq i \leq n$.

In the following, we use an example shown in [9] to illustrate the fuzzy risk analysis process of the proposed method. Assume that there are three manufactories C_1 , C_2 and C_3 and assume that the components A_1 , A_2 and A_3 are the same product manufactured by different manufactories, where component A_i is made by manufactory C_i consisting of three sub-components A_{i1} , A_{i2} and A_{i3} , where $1 \leq i \leq 3$.

Assume that there are two evaluating items, i.e., the probability of failure R_{ik} and the severity of loss W_{ik} , where R_{ik} and W_{ik} are generalized fuzzy numbers, $1 \leq i \leq 3$, and $1 \leq k \leq 3$. Evaluate each sub-component A_{ik} to derive the probability of failure R_i of component A_i made by manufactory C_i , where R_{ik} denotes the probability of failure of the sub-component A_{ik} , W_{ik} denotes the severity of loss of the sub-component A_{ik} , $1 \leq i \leq 3$ and $1 \leq k \leq 3$. We use a 9-number linguistic term set shown in Table 1 [16] to represent the linguistic terms and their corresponding generalized fuzzy numbers, where the linguistic terms shown in Table 1 are used to represent the linguistic values of

the evaluating items R_{ik} and W_{ik} of the sub-components A_{i1} , A_{i2} and A_{i3} made by each manufactory C_i , where $1 \leq i \leq 3$, as shown in Table 2 [5].

Table 1. Linguistic Terms and Their Corresponding Generalized Fuzzy Numbers [16]

Linguistic Terms	Generalized Fuzzy Numbers
Absolutely-Low	(0.0, 0.0, 0.0, 0.0; 1.0)
Very-Low	(0.0, 0.0, 0.02, 0.07; 1.0)
Low	(0.04, 0.10, 0.18, 0.23; 1.0)
Fairly-Low	(0.17, 0.22, 0.36, 0.42; 1.0)
Medium	(0.32, 0.41, 0.58, 0.65; 1.0)
Fairly-High	(0.58, 0.63, 0.80, 0.86; 1.0)
High	(0.72, 0.78, 0.92, 0.97; 1.0)
Very-High	(0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely-High	(1.0, 1.0, 1.0, 1.0; 1.0)

Table 2. Linguistic Values of the Evaluating Items of the Sub-Components Made by the Manufactories [5]

Manufactory	Sub-Components	Linguistic Values of the Severity of Loss	Linguistic Values of the Probability of Failure	Degree of Confidence of the Probability of Failure
C ₁	A ₁₁	\tilde{w}_{11} = Low	\tilde{R}_{11} = Fairly-Low	$w_{R_{11}} = 0.9$
	A ₁₂	\tilde{w}_{12} = Fairly-High	\tilde{R}_{12} = Medium	$w_{R_{12}} = 0.7$
	A ₁₃	\tilde{w}_{13} = Very-Low	\tilde{R}_{13} = Fairly-High	$w_{R_{13}} = 0.8$
C ₂	A ₂₁	\tilde{w}_{21} = Low	\tilde{R}_{21} = Very-High	$w_{R_{21}} = 0.85$
	A ₂₂	\tilde{w}_{22} = Fairly-High	\tilde{R}_{22} = Fairly-High	$w_{R_{22}} = 0.9$
	A ₂₃	\tilde{w}_{23} = Very-Low	\tilde{R}_{23} = Medium	$w_{R_{23}} = 0.9$
C ₃	A ₃₁	\tilde{w}_{31} = Low	\tilde{R}_{31} = Fairly-Low	$w_{R_{31}} = 0.95$
	A ₃₂	\tilde{w}_{32} = Fairly-High	\tilde{R}_{32} = High	$w_{R_{32}} = 0.8$
	A ₃₃	\tilde{w}_{33} = Very-Low	\tilde{R}_{33} = Fairly-High	$w_{R_{33}} = 1.0$

In Table 2, $w_{R_{ik}}$ denotes the degree of confidence of the decision-maker with respect to sub-components A_{ik} made by manufactory C_i , where $1 \leq i \leq 3$ and $1 \leq k \leq 3$. The process for fuzzy risk analysis is illustrated as follows:

[Step 1] The probability of failure R_1 of the component A_1 made by the manufactory C_1 is calculated by Eq. (9), shown as follows:

$$\begin{aligned} \tilde{R}_1 &= [\tilde{R}_{11} \otimes \tilde{W}_{11} \oplus \tilde{R}_{12} \otimes \tilde{W}_{12} \oplus \tilde{R}_{13} \otimes \tilde{W}_{13}] \oslash [\tilde{W}_{11} \oplus \tilde{W}_{12} \oplus \tilde{W}_{13}] \\ &= (0.1659, 0.2803, 0.7463, 1.1545; 0.7). \end{aligned}$$

In the same way, the probability of failure R_2 of the component A_2 made by the manufactory C_2 and the probability of failure R_3 of the component A_3 made by the manufactory C_3 can be calculated by Eq. (9), respectively, where

$$\begin{aligned} R_2 &= (0.3221, 0.4949, 1.1392, 1.6373; 0.85), \\ R_3 &= (0.3659, 0.5134, 1.1189, 1.5984; 0.8). \end{aligned}$$

[Step 2] Based on Eq. (2), we can transform the generalized fuzzy numbers R_1 , R_2 and R_3 into standardized generalized fuzzy numbers \tilde{R}_1^* , \tilde{R}_2^* and \tilde{R}_3^* , respectively, where

$$\tilde{R}_1^* = (0.083, 0.1402, 0.3732, 0.5773; 0.7),$$

$$\tilde{R}_2^* = (0.1611, 0.2475, 0.5696, 0.8187; 0.85),$$

$$\tilde{R}_3^* = (0.183, 0.2567, 0.5595, 0.7992; 0.8).$$

[Step 3] Based on Eq. (3), we can calculate the defuzzified values $x_{\tilde{R}_1^*}$, $x_{\tilde{R}_2^*}$ and $x_{\tilde{R}_3^*}$ of the standardized generalized fuzzy numbers \tilde{R}_1^* , \tilde{R}_2^* and \tilde{R}_3^* , respectively, where $x_{\tilde{R}_1^*} = 0.29$, $x_{\tilde{R}_2^*} = 0.45$ and $x_{\tilde{R}_3^*} = 0.45$.

[Step 4] Based on Eq. (4), we can calculate the spreads $STD_{\tilde{R}_1^*}$, $STD_{\tilde{R}_2^*}$ and $STD_{\tilde{R}_3^*}$ of the standardized generalized fuzzy numbers \tilde{R}_1^* , \tilde{R}_2^* and \tilde{R}_3^* , respectively, where $STD_{\tilde{R}_1^*} = 0.2271$, $STD_{\tilde{R}_2^*} = 0.3026$ and $STD_{\tilde{R}_3^*} = 0.2844$.

[Step 5] Based on Eq. (5), we can calculate the ranking values $Score(\tilde{R}_1^*)$, $Score(\tilde{R}_2^*)$ and $Score(\tilde{R}_3^*)$ of the standardized generalized fuzzy numbers \tilde{R}_1^* , \tilde{R}_2^* and \tilde{R}_3^* , respectively, where

$$Score(\tilde{R}_1^*) = 0.29 \times 0.7 \times (1 - 1.5 \times 0.2271) = 0.1338,$$

$$Score(\tilde{R}_2^*) = 0.45 \times 0.85 \times (1 - 1.5 \times 0.3026) = 0.2089,$$

$$Score(\tilde{R}_3^*) = 0.45 \times 0.8 \times (1 - 1.5 \times 0.2844) = 0.2064.$$

Because $Score(\tilde{R}_2^*) > Score(\tilde{R}_3^*) > Score(\tilde{R}_1^*)$, the ranking order of the risk of the manufactories C_1 , C_2 and C_3 is " $C_2 > C_3 > C_1$ ". It coincides with the result of the one presented in [5].

4. Conclusions

In this paper, we have presented a new method for ranking generalized fuzzy numbers. The proposed method considers three factors for ranking generalized fuzzy numbers, simultaneously, i.e., the defuzzified values, the height and the spreads of generalized fuzzy numbers. The proposed method gets better ranking results than Chen-and-Chen's method [3] and Chu-and-Tsao's method [10]. Moreover, we also apply the proposed ranking method to deal with fuzzy risk analysis problems. The proposed method provides a useful way for handling fuzzy risk analysis problems.

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