Variable Fuzzy Evaluation Model for the Water Resources Carrying Capacity in Jiangxi Province

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Abstract—Based on the data of nature, society, economics and water resources of Jiangxi province in 2009, we chose six indices which are the precipitation, irrigation ratio, the degree of water resource exploitation and utilization, per capita domestic water consumption, the per capita water supply and the ecological water use ratio to evaluate the water resources carrying capacity by means of a variable fuzzy evaluation model. The results show that the comprehensive grade is II for Jiangxi Province, where the current water exploitation and use has reached a relative high degree, and there is only a very limited water carrying capacity. The mode is easy and comfortable, which can be applied in the practice .

Keywords- water resources capacity; variable fuzzy evaluation model; Jiangxi province

I. INTRODUCTION

The concept of water resource carrying capacity was first suggested by the China Xinjiang Water Resource Soft-Science Research Panel. However, an explicit definition of this concept has not generally been given, either at home or abroad, until now. Some believe that the carrying capacity of a water resource is the ability of that resource to continually support a sound social system^[1], whereas others consider the concept to represent a threshold value for the resource's capacity to support human activity ^[2]. In this study, the carrying capacity of water resources are defined as the maximum amount of water required to support human activity in certain stages of social development, which can be borne by water resources under favorable eco-economic conditions. Previous researches on water resources carrying capacity mainly focused on comprehensive evaluation ^[3], system dynamic simulation ^[4] and multi-objective analysis. Some foreign scholars brought water resources carrying capacity into sustainable development theory ^[5]. However, there have been few attempts to study and understand the water resources carrying capacity of Jiangxi province. In this paper, we evaluate the carrying capacity of the water resources in Jiangxi province with a variable fuzzy evaluation model to provide decision makers with a basis upon which to draw up policies on the sustainable exploitation and utilization of these water resources.

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II. METHODS

A. Definition of variable fuzzy aggregation

Definition 1.1: Assume a fuzzy concept A of domain U to be in any point of the relative subjection function axis, $(u \in U) \cdot \mu_A(u)$ represents the relative subjection degree of u to A, which shows the attract ability, and $\mu_{A^c}(u)$ is the relative subjection degree of u to A^c , which shows the repellency^[6]. Let

$$D_{A}(u) = \mu_{A}(u) - \mu_{A^{c}}(u)$$
(1)

in which, $D_A(u)$ is the relative difference degree of u to A. Mapping

$$\begin{cases} D_A : D \to [-1, 1] \\ u \mid \to D_A(u) \in [-1, 1] \end{cases}$$
(2)

Eq. (2) shows the relative difference function of U to A. According to

$$\mu_A(u) + \mu_{A^c}(u) = 1 \tag{3}$$

then

$$D_A(u) = 2\mu_A(u) - 1$$
 (4)

or

$$u_A(u) = (1 + D_A(u))/2$$
 (5)

Definition 1.2: Let

$$V = \left\{ (u, D) \mid u \in U, D_{A}(u) = \mu_{A}(u) - \mu_{A^{c}}(u), D \in [-1, 1] \right\}$$
(6)

$$A_{+} = \mu \mid u \in U, 0 < D_{A}(u) \le 1$$
 (7)

$$A_{-} = \left\{ u \mid u \in U, -1 \le D_{A}(u) < 0 \right\}$$

$$(8)$$

$$A_0 = \left\{ u \mid u \in U, D_A(u) = 0 \right\}$$
(9)

where V is fuzzy variable aggregation; A_{+} , A_{-} , and A_{0}

represent the attraction domain, the repulsion domain, and the gradual change border, respectively.

Definition 1.3: Assume C to be the variable gene aggregation of V, which can be described with the following formula:

$$C = \left\{ C_A, C_B, C_C \right\} \tag{10}$$

Where C_A is the variable model aggregation, C_B is the

variable model parameter aggregation, and C_C is the variable aggregation of genes other than the model and parameter. Suppose

$$A^{-} = C(A_{+}) = \left\{ u \mid u \in U, \ 0 < D_{A}(u) \le 1, \ -1 \le D_{A}(C(u)) < 0 \right\}$$
(11)

$$A^{+} = C(A_{-}) = \left\{ u \mid u \in U, -1 \le D_{A}(u) < 0, 0 \le D_{A}(C(u)) \le 1 \right\}$$
(12)

 $V\,$ is the variable domain of fuzzy variable aggregation

concerning variable gene aggregation. Suppose

$$A^{(+)} = C(A_{(+)}) = \left\{ u \mid u \in U, 0 < D_A(u) \le 1, 0 < D_A(C(u)) \le 1 \right\}$$
(13)

$$A^{(-)} = C(A_{(-)}) = \left\{ u \mid u \in U, -1 \le D_A(u) < 0, -1 \le D_A(C(u)) < 0 \right\}$$
(14)

Where $A^{(+)}$ and $A^{(-)}$ are the variable domains of variable fuzzy sets V about the alterable factors aggregate C.

B. Relative difference function

When $X_0 = [a, b]$, the attraction domain of fuzzy variable aggregation is in the real axis, and $0 < D_A(u) \le 1$. X = [c, d] is the interval from up to down, including $X_0(X_0 \subset X)$.

According to the definition of variable fuzzy aggregation V, it is obvious that [c,d] and [a,b] are both regions of rejection, excluding the domain of V, namely $-1 \le D_A(u) < 0$. Suppose M is the point value of $D_A(u) = 1$ in the attraction domain [a,b], then M is not always the midpoint value of region [a,b] according to physical analysis.

When x is the value of any point in the X interval, and x is to the left of M, the relative difference function model can be described as follows:

$$\begin{cases} D_A(u) = \left(\frac{x-a}{M-a}\right)^{\beta}; x \in [a, M] \\ D_A(u) = -\left(\frac{x-a}{c-a}\right)^{\beta}; x \in [c, a] \end{cases}$$
(15)

When x is to the right of M, the relative difference function model can be represented as follows:

$$\begin{cases} D_{A}(u) = (\frac{x-b}{M-b})^{\beta}; x \in [M,b] \\ D_{A}(u) = -(\frac{x-b}{d-b})^{\beta}; x \in [b,d] \\ D_{A}(u) = -1; x \notin (c,d) \end{cases}$$
(16)

In Eqs. (15) and (16), β is a non-negative index, and generally $\beta = 1$, so the relative difference function is a linear function. Eqs. (15) and (16) satisfy: (1) when x = a, x = b, $D_A(u) = 0$; (2) when x = M, $D_A(u) = 1$; (3) when x = c, x = d, $D_A(u) = -1$. After determining $D_A(u)$, the relative subjection degree $\mu_A(u)$ can be calculated according to Eq. (5).

C. Variable fuzzy evaluation model

Suppose there are n swatch aggregates that can describe the conditions of the water resource carrying capacity, as follows:

$$X = \{x_1, x_2, \dots x_n\}$$
(18)

The characteristics of swatch j can be denoted by the eigenvalue of m indices.

$$x_{j} = (x_{1j}, x_{2j}, \dots x_{mj})^{T}$$
(19)

Then, the swatch aggregate can be denoted by matrix $m \times n$ index eigenvalue

$$X = (x_{ii}) \tag{20}$$

Where x_{ij} is the index eigenvalue of swatch j and index i; i = 1, 2, ..., n. The swatch aggregate can be identified according to the m indices and the c eigenvalues, then the $m \times c$ matrix is as follows:

$$Y = (y_{ih}) \tag{21}$$

Where y_{ih} is the standard eigenvalue of level h and index i, h = 1,2,...c.

The attraction domain and the bound domain matrix of variable aggregation about the water resources carrying

capacity in any place can be determined by referring to the index standard matrix

$$I_{ab} = ([a_{ih}, b_{ih}])$$
(22)
$$I_{cd} = ([c_{ih}, d_{ih}])$$
(23)

According to c levels of water resources carrying

capacity, the matrix of M_{ih} can be represented as follows:

$$M = M_{ih} \tag{24}$$

It is necessary to determine whether the swatch eigenvalue x_{ij} is to the left or right of M_{ih} according to Eqs. (22–24), then the difference degree $D_A(x_{ij})_h$ can be calculated by Eqs. (15–17), and the relative subjection degree $\mu_A(x_{ij})_h$ matrix of level h can be calculated according to Eq. (5).

$$[U_{h}] = (\mu_{A}(x_{ij})_{h})$$
(25)

The fuzzy evaluation model cited in the references will be applied.

$$ju_{h}' = \frac{1}{1 + \left\{ \frac{\sum_{i=1}^{m} \left[w_{i} (1 - \mu_{A}(x_{ij})_{h}) \right]^{p}}{\sum_{i=1}^{m} w_{i} \mu_{A}(x_{ij})_{h} \right\}^{p}} \right\}^{a/p}$$
(26)

Where ju_h' stands for the comprehensive relative subjection degree that is not unitary; α is the rule parameter of model optimization; w_i is the index weight; m is the identification index number; p is the distance parameter, with p = 1 and p = 2 representing the Hamming distance and Euclid distance, respectively.

The comprehensive relative subjection degree matrix that is not unitary can be calculated from Eq. (26)

$$U' = (ju_h') \tag{27}$$

(28)

The comprehensive relative subjection degree matrix that is unitary can be described as follows:

 $U = (ju_h)$

Where

$$ju_h = ju_h' / \sum_{h=1}^c ju_h'.$$
 (29)

According to the inapplicability of the greatest subjection degree of the fuzzy concept under the classification condition, the swatch can be evaluated according to the level eigenvalue put forward in the paper (Chen. 1998).

$$H = (1, 2, ..., c) \bullet U \tag{30}$$

III. CASE ANALYSIS

General situation of the Jiangxi province

Α.

Jiangxi Province is located in the middle and lower reaches of Yangtze River, and the average annual Precipitation is about 1640 mm, the average annual total water resources quantity is 1.42×10^9 m³. However, the water resources are not well balanced due to the seasonal and regional disparity. In the regional disparity, the quantity of water resources in north is more than in south, and the he average annual Precipitation is about 1400-1900 mm^[7]; In the seasonal disparity, most precipitation focus on February, May and June, which Approximately composes the whole year precipitation 45%-50%. Three problems exist in Jiangxi province simultaneously which are flood disaster, water shortage, and water environment deterioration. Therefore, the paper considers that research on the water resources sustainable utilization of Jiangxi province has very important significance in improving the harmonious development of economics, society and ecology.

B. Selection and gradation of evaluation factors

The carrying capacity of a water resource is a concept with double attributes, involving both nature and society. Obviously, this means that the system is complex and large scale, with numerous contributing factors, such as population, resources, the environment, ecology, society, economy, technology, etc. Based on the exploitation and utilization situation of water resources in the Jiangxi province and consulting the indices system of water supply and demand in China ^[8-9], we chose six indices (Table 1) to evaluate the water resources carrying capacity of the Jiangxi province.

These six evaluation factors are precipitation(u_1), the irrigation ratio (u_2 : the ratio of the irrigation area to the plantation area); the degree of water resource exploitation and utilization (u_3 : the ratio of water supply to gross water resources); per capita domestic water consumption(u_4 : the ratio of the consumption of domestic water to the gross population); the per capita water supply (u_5 : the ratio of the water supply to the gross population); and the ecological water use ratio (u_6 : the ratio of the ecological water use to the total water use).

Consulting other evaluation standards for water resources, we defined three grades for the significance of six evaluation factors to the carrying capacity of the water resources in the Jiangxi province^[10-11]. The indices of each grade are listed for each factor (Table 2). V_1 represents for the best situation, which shows that the water resources still have great potential, and the degree of water resource utilization and the scale of development are small. V_3 represents the worst situation, in which the water resource carrying capacity is close to full and more exploitation of the water resource will lead to a shortage in the water resource

Administrative division	Precipitation/mm	irrigation ratio/%	the degree of water resource exploitation and utilization /%	per capita domestic water consumption (L/person d)	the per capita water supply (m ³ / person)	The ecological water use ratio/%	
Nanchang city	1179.8	76.69	68.55	133	719	7.16	
Jindezhen city	1526.7	61.7	20.49	126.9	529.2	0.19	
Pingxiang city	1551.6	57.9	21.42	119.2	434.5	0.24	
Jiujiang city	1201.8	60.8	19.15	121.2	501.1	0.13	
Xinyu city	1525.0	55.2	35.46	126.7	713.2	0.39	
Yintan city	1619.3	58.3	17.41	122.1	580.2	0.11	
Ganzhou city	1329.2	72.0	15.70	120.6	385.8	0.11	
Yichun city	1469.5	65.5	30.52	120.0	805.1	0.12	
Shang rao city	1565.0	50.5	14.25	112.6	451.4	0.06	
Jian city	1294.3	90.6	25.74	115.6	792.5	0.09	
Fuzhou city	1506.8	57.2	16.38	118.1	615.3	0.08	
Jiangxi province	1392.0	65.3	22.02	120.6	579.8	0.41	

TABLE 1.STATISTICS OF EVALUATION INDICS IN JIANGXI PROVINCE

TABLE 2 INDICS OF THE GRADATION OF EACH EVALUATION FACTORS

Evaluation factors	V_1	V_2	V_3
Precipitation/mm	>450	375	<300
irrigation ratio /%	<20	40	>60
he degree of water resource exploitation and utilization/%	<30	50	>70
per capita domestic water consumption (L/person d)	<70	100	>130
the per capita water supply (m ³ /person)	>4500	3500	<2500
the ecological water use ratio /%	>5	3	<1

and environmental deterioration; V_2 is between V_1 and V_3 , where the scales of water resource exploitation and utilization have reached a certain level, but there is still potential for further exploitation and utilization.

C. Calculation of a comprehensive evaluation According to Table 2, the status index eigenvalue and the index standard value matrix can be calculated as follows: $X = \begin{bmatrix} 1179.8 & 1526.7 & 1551.6 & 1201.8 & 1525.0 & 1619.3 & 1329.2 & 1469.5 & 1565.0 & 1294.3 & 1506.8 & 1392.0 & 1566.6 & 61.7 & 57.9 & 60.8 & 55.2 & 58.3 & 72.0 & 65.5 & 50.5 & 90.6 & 57.2 & 65.3 & 68.55 & 20.49 & 21.42 & 19.15 & 35.46 & 17.41 & 15.70 & 30.52 & 14.25 & 25.74 & 16.38 & 22.02 & 133 & 126.9 & 119.2 & 121.2 & 126.7 & 122.1 & 120.6 & 120.0 & 112.6 & 115.6 & 118.1 & 120.6 & 719 & 529.2 & 434.5 & 501.1 & 713.2 & 580.2 & 385.8 & 805.1 & 451.4 & 792.5 & 615.3 & 579.8 & 7.16 & 0.19 & 0.24 & 0.13 & 0.39 & 0.11 & 0.111 & 0.12 & 0.06 & 0.09 & 0.08 & 0.41 & 14.56 & 14.$

$$Y = \begin{bmatrix} 450 & 450 \sim 300 & 300 \\ 20 & 20 \sim 60 & 60 \\ 30 & 30 \sim 70 & 70 \\ 70 & 70 \sim 100 & 130 \\ 4500 & 4500 \sim 2500 & 2500 \\ 5 & 1 \sim 5 & 5 \end{bmatrix} = (y_{ih})$$

Where i = 1, 2, ..., 6 indicates the index number; j = 1, 2, ..., 12 is the subsection mark; and h = 1, 2, 3 is the level number. Consulting the index standard value matrix Y and the actual situation of the Jiangxi province, the attraction domain matrix, bound matrix, and point value M_{ih} can be determined as follows:

$$I_{ab} = \begin{bmatrix} [750, 450] & [450, 300] & [300, 0] \\ [0, 20] & [20, 60] & [60, 100] \\ [0, 30] & [30, 70] & [70, 110] \\ [0, 70] & [70, 100] & [100, 160] \\ [7000, 4500] & [4500, 2500] & [2500, 0] \\ [9, 5] & [5, 1] & [1, 0] \end{bmatrix}$$
$$I_{cd} = \begin{bmatrix} [750, 300] & [450, 0] & [300, 0] \\ [0, 60] & [0, 100] & [20, 100] \\ [0, 70] & [0, 110] & [30, 110] \\ [0, 100] & [0, 160] & [70, 160] \\ [7000, 2500] & [4500, 0] & [2500, 0] \\ [9, 1] & [5, 0] & [1, 0] \end{bmatrix}$$

	750	450	0]
	0	20	100
М	0	30	110
MI =	0	70	160
	7000	4500	0
	9	5	0

The position of x_{ij} , calculated according to I_{ab} , I_{cd} , and M, is to the left or right of M_{ih} . The difference degree $D_A(x_{ij})_h$ can be calculated with Eq. (15) or (16), and the relative subjection degree $\mu_A(x_{ij})_h$ can be determined with Eq. (17). When $j = 1, 2, \dots, 6$, the relative subjection degree matrix corresponding to h = 1, 2, 3 is as follows:

The unitary vectors can be obtained with an analytic hierarchy Process^[12] and expert opinion, as follows:

$$w = [0.2, 0.15, 0.2, 0.15, 0.2, 0.1]$$

With the variable fuzzy evaluation model of Eq. (26), the relative subjection degree of the water resources carrying capacity at different levels in the Jiangxi province can be calculated.

From $[U_2]$, the relative subjection degree vector with j = 3 can be calculated as

follows: $[(\mu_2)]_3 = (0, 0.362, 0.327, 0.05, 0.473, 0.256)$ Let the distance parameter p = 1, the model optimization principle parameter $\alpha = 2$, so when j = 3, h = 2, the variable fuzzy valuation model of Eq. (26) can be expressed as follows:

$$3u_{2}' = \frac{1}{1 + \left\{\frac{\sum_{i=1}^{8} \left[w_{i}(1 - \mu_{A}(x_{i3})_{2})\right]}{\sum_{i=1}^{8} w_{i}\mu_{A}(x_{i3})_{2})}\right\}^{2}}$$

In the above formula, $3u_2' = 0.010$ can be calculated in the same way. When h = 1, 2, 3, the relative membership degree is as follows:

3u' = [0.845, 0.010, 0.145]

When $j = 1, 2, \dots, 12$, the relative subjection degree that is not unitary for the water resources carrying capacity can be obtained as follows

Using Eq. (30), the level eigenvalue vector of the water resources carrying capacity can be calculated as follows:

0.149 0.142 0.145 0.344 0.300 0.352 0.319 0.347 0.459 0.371 0.038 0.331

$H = (1, 2, 3) \bullet$

0.792 0.299 0.845 0.644 0.681 0.640 0.590 0.651 0.540 0.621 0.614 0.611 0.059 0.559 0.010 0.012 0.019 0.008 0.091 0.002 0.001 0.008 0.008 0.057 0.149 0.142 0.145 0.344 0.300 0.352 0.319 0.347 0.459 0.371 0.038 0.331 = (1.357, 1.773, 1.3, 1.7, 1.619, 1.712, 1.729, 1.696, 1.919,1.75, 0.744, 1.718)

Distrcts	Nan chang	Jinde zhen	Ping xiang	Jiu jiang	Xin yu	Yin tan	Gan zhou	Yi chun	Shang rao	Jian	Fu zhou	Jiangxi
Eigenvalue	1.357	1.773	1.3	1.7	1.619	1.712	1.729	1.696	1.919	1.75	0.744	1.718
Assessment grade	1	2	1	2	2	2	2	2	2	2	1	2

D. Results analysis

From these results (Table 4), we can see that the evaluation grade for the water resources carrying capacity of Nanchang, Jindezhen, Jiujiang, Xinyu, Yintan, Ganzhou, Yichun, Shangrao, Jian and Jiang xi province is 2, and that the current water exploitation and utilization have reached relatively high levels, although there is still some exploitation potential; However, the situations of Nanchang, Pingxiang and Fuzhou are more positive, with assessment grades of 1. It is obvious that the water resource potential of these three districts is relatively high, and that the water resources can still sustain future economic development and environmental protection. The sequence of water resources carrying capacity is respectively Fuzhou, Pingxiang, Nanchang, Xinyu, Yichun, Jiujiang, Yintan, Ganzhou ,Jian,Jingdezhen,Shangrao. Overall, it is necessary to strengthen the integrated management of the water resources to utilize them scientifically and sustainably.

IV. CONCLUSIONS

This study evaluated the carrying capacity of the water resources of Jiangxi province using a variable fuzzy evaluation model. Our results show that the carrying capacity of most districts has an evaluation grade of 2, and that the current water exploitation and utilization in Jiangxi province have reached relatively high levels, although there is still some exploitation potential. Water resources are the most important limiting factor in the socio-economic development and the construction of an ecologically sound environment in any region. Therefore, it is strategically important to reduce the water resources crisis in Jiangxi province and to alleviate the conflict between ecology and the economic exploitation of these resources.

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^{0.792} 0.299 0.845 0.644 0.681 0.640 0.590 0.651 0.540 0.621 0.614 0.611 $U = \begin{vmatrix} 0.059 & 0.559 & 0.010 & 0.012 & 0.019 & 0.008 & 0.091 & 0.002 & 0.001 & 0.008 & 0.008 & 0.057 \end{vmatrix}$