	y_1	y_2	y_3	y_4
Least squares method				
Original data	20 ± 3	10 ± 2	28 ± 4	16 ± 3
Reconciliated values	18	10	28	18
Reconciliated fuzzy intervals				
Triangular fuzzy intervals	$(17, 20, 23)$	$(8, 10, 12)$	$(24, 28, 32)$	$(13, 16, 19)$
$\alpha* : \frac{1}{3}$				
Reconciliated cores	$[18, 18]$	$[8 + \frac{2}{3}, 12 - \frac{2}{3}]$	$[26 + \frac{2}{3}, 29 + \frac{1}{3}]$	$[18, 18]$
Reconciliated supports	$[17, 19]$	$[8, 12]$	$[25, 31]$	$[17, 19]$
Reconciliated cores: 2d round	$[18, 18]$	$[10, 10]$	$[28, 28]$	$[18, 18]$

Table 2: Reconciliated flows For Example 2

generation process and not as an additional constraint. In contrast one could consider reconciling variances as a data fusion problem.

Mind that minimizing a sum of absolute valued deviations (and to a lesser extent quadratic), runs the risk of making certain values of x_i deviate significantly from the data $\hat{x}_i$, whereas the max-min approach is designed to keep all of them as close as possible to the initial data. The latter approach seems to be more reasonable if the data come as single estimate from an expert or other sources for each quantity. This approach is currently being studied for analyzing the material flow of rare earth elements in the anthroposphere of the EU-27.

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References

[1] F. Benhamou, L. Granvilliers, F. Goualard. Interval Constraints: Results and Perspectives. *New Trends in Constraints*, LNAI 1865, pages 1-16, Springer, 2000. *22nd European Symposium on Computer Aided Process Engineering* (I.D. Lockart Bogle, M. Fairweather, Eds.), Elsevier 2012, 122-126

[2] P.H. Brunner and H. Rechberger (2004). Practical Handbook of Material Flow Analysis. Lewis Publishers.

[3] T. Calvo, G. Beliakov Aggregation functions based on penalties, *Fuzzy Sets and Systems*, 161(10), 2010, 1420-1436.

[4] D. Dubois Possibility theory and statistical reasoning *Comput. Stat. & Data Anal.*, 51, 47-69, 2006

[5] D. Dubois, H. Fargier, H. Prade. Possibility theory in constraint satisfaction problems: Handling priority, preference and uncertainty. *Applied Intelligence*, 6, 287-309, 1996.

[6] D. Dubois, P. Fortemps. Computing improved optimal solutions to max-min flexible constraint satisfaction problems. *Eur. J. of Operation Research*, 118, 95-126, 1999.

[7] Dubois D. Foulloy L. Mauris G. Prade H. Probability-possibility transformations, triangular fuzzy sets, and probabilistic inequalities. *Reliable Computing*. 2004. 10, 273-297

[8] D. Dubois, E. Kerre, R. Mesiar, H. Prade. Fuzzy interval analysis. In: *Fundamentals of Fuzzy Sets*, Dubois, D. Prade, H., Eds: Kluwer, Boston, Mass, The Handbooks of Fuzzy Sets Series, 483-581, 2000.

[9] D. Dubois, S. Moral, H. Prade. A semantics for possibility theory based on likelihoods. *J. of Math. Anal. Appl.*, 205, 359-380, 1997.

[10] D. Dubois, H. Prade, Philippe Smets: A definition of subjective possibility. *Int. J. Approx. Reasoning* 48(2): 352-364 (2008)

[11] L. Granvilliers, F. Benhamou. Algorithm 852: RealPaver: an interval solver using constraint satisfaction techniques. *ACM Trans. on Mathematical Software* 32(1), 138-156, 2006.

[12] L. Jaulin, M. Kieffer, O. Didrit, E. Walter. *Applied Interval Analysis*, Springer, London, 2001.

[13] O. Lhomme. Consistency Techniques for Numeric CSPs. *Proc. Int. Joint Conf on Artificial Intelligence*(IJCAI 1993), pages 232-238.

[14] G. Mauris: Expression of Measurement Uncertainty in a Very Limited Knowledge Context: A Possibility Theory-Based Approach. *IEEE T. Instrum. and Meas.* 56(3): 731-735 (2007)

[15] G. Mauris: Possibility distributions: A unified representation of usual direct-probability-based parameter estimation methods, *Int. J. of Approx. Reasoning*, 52(9), 1232-1242 (2011).

[16] S. Narasimhan, C. Jordache, Data reconciliation and gross error detection: an intelligent use of process data, Gulf Publishing Company, Houston, 2000.

[17] H. Steyaert, F. Van Parys, R. Baekeland and E. Kerre. Implementation of piecewise linear fuzzy quantities, *Int. J. Intelligent Systems*, 10, 1049-1059, 1995.

[18] S. M. Stigler. *The History of Statistics: The Measurement of Uncertainty before 1900.* Belknap Press/Harvard University Press, 1990.