**Theorem 4.7.** *Let* $(\Omega, \mathbb{A}, P)$ *and* $(\Xi, \mathbb{B}, Q)$ *be classical probability spaces and let* $(\Omega, \mathcal{M}(\mathbb{A}), \int(\cdot)\,\mathrm{d}P)$ *and* $(\Xi, \mathcal{M}(\mathbb{B}), \int(\cdot)\,\mathrm{d}Q)$ *be the corresponding fuzzy probability spaces. Let* $h_c$ *be a classical observable. Then there exists a unique fuzzy observable* $h$ *such that* $h_c(B) = h(B)$ *for all* $B \in \mathbb{B}$.

Denote RFP the subcategory of FP having the fuzzy probability spaces as objects and the restricted fuzzy observables as morphisms.

The next theorem follows directly from the previous one.

**Theorem 4.8.** *The categories CP and RFP are isomorphic.*

ANSWER. There is a canonical isomorphism between CP, representing the classical probability theory, and the subcategory RFP of FP, representing the fuzzy probability theory. The objects of the two categories are in a canonical one-to-one correspondence, but the fuzzy probability theory has "more" morphisms. Indeed, to each classical domain of probability $\mathbb{A}$ ($\sigma$-field of sets) there corresponds a unique domain of fuzzy probability theory $\mathcal{M}(\mathbb{A})$ (the set of all measurable functions ranging in $I$) but, in general, there are fuzzy observables of $\mathcal{M}(\mathbb{B})$ to $\mathcal{M}(\mathbb{A})$ which are not extensions of any classical random variable from $\mathbb{B}$ to $\mathbb{A}$.

The author is indebted to the referee for his valuable suggestions how to improve the original text.

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