Reflections on a fuzzy logic of scientific discovery and fuzzy structures of scientific revolutions

Rudolf Seising[1]

[1] *European Centre for Soft Computing*
Edificio de Investigación,
Calle Gonzalo Gutiérrez Quirós S/N
33600 Mieres, Asturias
rudolf.seising@softcomputing.es

Abstract

Popper's *Logic of Scientific Discovery* and Kuhn's *Structures of Scientific Revolutions* are enormous important concepts on theories and their dynamic in 20th century history and philosophy of science. How-ever, the works of the two have been criticized in the last half century and new views on theory nets and theory evolution appeared. In the last decades also new scientific methods and tools showed up and we can use them to describe the historical development of elements and networks of scientific theories. In this paper we make such use of Fuzzy Set Theory. We wonder: are Fuzzy Sets appropriate to describe scientific theories and their relationships to each other and also in time? Is there a fuzzification of the structures in philosophy of science? – This last question refers to the so-called "structuralist view" or "Metastructuralism" of theories that uses usual set theory. In this paper we propose a fuzzy approach to philosophy of science, that we name "fuzzy structuralist view" of scientific theories or "Fuzzy-Metastructuralism".

Keywords: Philosophy of science, (Meta)structuralism, fuzzy sets, Fuzzy structuralism

1. Introduction

The title of this contribution bears on two famous books: Sir Karl R. Popper's *The Logic of Scientific Discovery* [1] and Thomas S. Kuhn's *The Structure of Scientific Revolutions* [2].

Philosophy of science that appears to us in this paper is concerning scientific explanations of real systems and phenomena. Scientists observe these real systems and phenomena in natural environments and laboratories. They determine functions that represent the real system's properties and variables that characterize these systems. Scientists measure the values of the observed variables (observables) and therefore they collect dozens of data. Finally, scientists connect these real systems and phenomena with theoretical structures. They create these structures to have a "mapping" from the real world to the world of logics and mathematics. In this theoretical "paradise" they can formulate mathematical constants and variables, axioms and laws that represent the real systems and phenomena. Scientists suppose that there is a connection between the real world and the logical-mathematical world – otherwise it doesn't make sense to speak about empirical science.

However, nobody can be sure that a scientific theory is true. In the 1930s, the Austrian-British philosopher Karl R. Popper (1902-1994) established the critical rationalism rejecting this classical empiricism. In principle, scientific theories are always tentative, and subject to corrections or inclusion in a yet wider theory [1].

Later, the US-American historian and philosopher of science Thomas S. Kuhn (1922-1996) argued that there is no linear accumulation of new knowledge in the development of science. Science undergoes periodic revolutions and there are "paradigm shifts" in history of science, in which the nature of scientific inquiry within a particular field is transformed.

Kuhn claimed that there are three different stages of science: Prescience lacks a central paradigm. Later, when scientists attempt to enlarge the central paradigm by "puzzle-solving" prescience is followed by normal science. Normal science reaches a crisis when anomalous results build up. At this point a new paradigm can emerge, which subsumes the old results along with the anomalous results into one framework. This new paradigm is termed revolutionary science [2].

Subsequently a new approach to philosophy of science appeared with the work of the American mathematician and philosopher Patrick Suppes (born 1922), the US-American physicist Joseph D. Sneed (born 1938) and the Austrian philosopher Wolfgang Stegmüller (1923-1991): the so-called "Metastructuralism" or the "structuralist view of theories". In the last decades of the 20th century, Stegmüller, the Mexican-German philosopher C. Ulises Moulines, the German philosopher Wolfgang Balzer, and others developed this view into a framework intended to analyze networks of theories and their evolution [3-6, 8, 9]. This approach bases on informal logic and set theory. Ordinary sets represent the structures in a theoretical area (we will later name it the "theoretical layer") and also the structures in an real area ("real layer").

In this paper we propose to generalize this metastructuralist approach in philosophy of science by fuzzy sets: Fuzzy sets are a new concept in mathematics and also a new concept in science – a concept that forgoes precision. This can be regarded as an advantage – especially in connection with nonclassical scientific theories. In this paper attention will focus on considerations pertaining to the connection between empirical systems and theoretical structures and an intermediate layer of fuzzy structures to establish a "fuzzy approach" to the Metastructuralism in philosophy of science.

2. On philosophy of science

2.1. Some prehistory of philosophy of science

Philosophy of science reflects the basis of science, their assumptions and implications, their methods and results, their theories and experiments. We can distinguish between the philosophy of astronomy and physics, chemistry, and other empirical sciences, and we can also be interested in philosophies of social sciences and the humanities. However, these differing philosophies of scientific disciplines arose in differing historical periods and the earliest philosophical reflections on modern science started with theories and experiments in mechanics in the 17^{th} century. Two main views in philosophy of science arose in about the same period: The philosophical view of *rationalism* came to fundamental, logical and theoretical investigations using logics and mathematics to formulate axioms and laws whereas the view of *empiricism* paved the way for experiments to find or prove or refute natural laws. In both directions – from experimental results to theoretical laws or from theoretical laws to experimental proves or refutations – scientists have to bridge the gap that separates theory and practice in science.

From the empiricist point of view the source of our knowledge is sense experience. The English philosopher John Locke (1632-1704) used the analogy of the mind of a newborn as a "tabula rasa" that will be written by the sensual perceptions the baby has later. In Locke's opinion this perceptions provide information about the physical world. Locke's view is called "material empiricism" whereas the so called idealistic empiricism was hold by George Berkeley (1684-1753) and David Hume (1711-1776), an Irish and a Scottish philosopher: there exists no material world, only the perceptions are real.

This epistemological dispute is of great interest for historians of science but it is ongoing till this day and therefore it is of great interest for today's philosophers of science, too. Searching a bridge over the gap between rationalism and empiricism is a slow-burning stove in the history of philosophy of science.

2.2. 20th century's philosophy of science

Popper's *Logic of Scientific Discovery* was published already in 1934 in German but it became not influential before the English edition appeared in 1959. This work is a milestone in History of Philosophy of Science and it heralded a shift in differentiating between science and non-science, metaphysics or pseudo-science. In the "pre-Popper-times" philosophers tried to fix this demarcation in scientific language but in Popper's metatheory, named "Critical Rationalism", the decision of what is science and what is not science is related to theories and methods in these fields and not in the precision of the terms of language. Popper created this alternative concept to that of the Vienna Circle and the other Logical Empiricists who tried to analyze the constitution or the structure of scientific theories by using modern logic. Particularly the German philosopher Rudolf Carnap (1891-1970), who later was a professor in the United States of America, wrote in 1928 *The Logical Structure of the World* [10]. For Carnap and many others theories are sets of propositions and these propositions are built from data via induction. – Popper said: On the contrary! For Critical Rationalists scientific theories are not built from data by induction! There is no logical way from data to theory! Theories are hypotheses or conjectures and scientists test these hypotheses in experiments with intent to refute them. Even a great number of positive test results cannot confirm a scientific theory, but if there is only one outcome that is negative, this one counterexample shows that the theory is falsified. However, we can try to falsify our hypothesis and if we find one counterexample, then the hypothesis is refuted. Thus, in Critical Rationalism the falsifiability is the criterion of demarcation between what is scientific and what is not.

Another argument against the Logical Empiricism is the following: It seems very clear that we cannot reduce all our knowledge to sensual data. Therefore, we need so-called *theoretical elements* in addition to the empirical ones. These additional elements are being understood only in the context of a theory. They are more abstract, they are more distant from our perceptions than observational terms. To factor these elements in Logical Empiricism Carnap and the German philosopher Carl G. Hempel (1905-1997)[1] introduced in the 1950s the so-called "double language model". [11, 12] Whereas observational and therefore non-theoretical terms are elements of the observation language, theoretical terms are elements of the theoretical language. Later, also the US-American philosopher Willard van Orman Quine (1908-2000) criticized the empiricist differentiation between "analytical" and "synthetical". In short, the Logical Empiricism collapsed.

In 1962 Thomas Kuhn published *The Structure of Scientific Revolutions*. Later he exemplified that the idea to this book went back to 1947 when he was asked as a graduate student at Harvard University to teach a science class for humanities undergraduates on historical case studies.

In this book he criticized Popper's view on theory dynamics in science. As he could show in many cases of his historical research work, no replacement of a theory by another happened because of falsification [13, 2]. In his new view theory change in science is not a rational process and therefore we need assistance from sociology and psychology to explain the paths of science through history.

Kuhn's historical research convinced him that there were periods of "prescience" that lack any theory or paradigm, then there were periods of "normal science" with paradigm monopole and finally there were times of crisis that triggered "scientific revolutions". Most scientists in most periods have been "normal scientists". They are involved with puzzle-solving. Only if there

[1] He emigrated to Belgium (1934) and the USA (1937).

were many anomalies in opposition to the current paradigm a crisis appeared and a scientific revolution could happen. Later, Kuhn introduced the notion "disciplinary matrix" to replace "paradigm" because of many criticisms for having used the notion "paradigm" extremely loosely.

In the 1960s the controversy between Popper's view and Kuhn's view was discussed by almost all philosophers of science and it culminated in a conference at Bedford College in London in 1965 that was organized by the Hungarian philosopher Imre Lakatos (1922-1974)[2] to debate on their contradictory theories [14]. These discussions opened the door for many new developments in history and philosophy of science, e.g the view of "research programmes" by Lakatos' [15], the view of "research traditions" by Larry Laudan (born 1941) [16, 17], and the so-called "epistemological anarchism" by Paul Feyerabend (1924-1994) [18]. This is not the place to follow all these directions but we will turn to a then also new established trend in obtaining systematic rational reconstructions of scientific theories.

2.3. Metastructuralism in philosophy of science

Two trends in obtaining systematic rational reconstructions of empirical theories can be found in the philosophy of science in the latter half of the 20th century: the Carnap approach (after Rudolf Carnap) and the Suppes approach (after Patrick Suppes). In both, the first step consists of an axiomatization that seeks to determine the mathematical structure of the theory in question. However, whereas in the Carnap approach the theory is axiomatized in a formal language, the Suppes approach uses informal set theory. Thus, in the Suppes approach, one is able to axiomatize real physical theories in a precise way without recourse to formal languages. This approach can be traced back to Suppes' proposal in the 1950s to include the axiomatization of empirical theories of science in the metamathematical programme of the French group *Bourbaki* [19].

Later, in the 1970s, Joseph D. Sneed developed informal semantics to include not only mathematical aspects, but also application subjects of scientific theories in this framework, based on this method. In his book [8], Sneed presented the view that all empirical claims of physical theories have the form "*x* is an *S*", where "is an *S*" is a set-theoretical predicate (e.g., "*x* is a classical particle mechanics"). Every physical system that fulfils this predicate is called a model of the theory, say *T*. For example, the class *M* of a theory's models is characterized by empirical laws that consist of conditions governing the connection of the components of physical systems. Therefore, we have models of a scientific theory, and by removing their empirical laws, we get the class $M_p(T)$ of so-called potential models of the theory.

These potential models of the theory consist of *theoretical terms*. The meaning of such a theoretical term becomes determined through the axioms of ***T.*** The meaning of the term 'force', for example, is seen to be determined by Newton's laws of motion and further laws about special forces, such as the law of gravitation.

In Carnap's approach a scientific theory is an interpreted axiomatic formal system and in his book *Philosophical Foundations of Physics* [7] he distinguished between observational and theoretical terms. This view was based on the distinction between two kinds of scientific laws, namely empirical laws and theoretical laws.

In Carnap's view empirical observations can directly confirm empirical laws and they deal with measurable physical quantities. But there are other objects or properties that we cannot observe or measure but we can only *infer* them from direct observations. Theoretical laws are concerned with these objects or properties. We cannot justify them by means of direct observation. Thus, theoretical laws are not an inductive generalizations but hypotheses reaching beyond experience. Therefore, we have to emphasize the important difference between empirical and theoretical laws: empirical laws can explain and forecast facts, whereas theoretical laws can explain and forecast empirical laws. To justify a theoretical law is not to test the law itself but to test the empirical laws that are among its consequences.

Carnap's distinction between empirical and theoretical laws led him distinguish between observational and theoretical terms. In some situations the borderline seems to be clear, e.g. the laws of kinetic gas theory are empirical and in quantum mechanics we see theoretical laws, but this was not always the case. Sometimes this distinction corresponds to that between macroscopic and microscopic or sub-atomic phenomena.

Five years later, in his book [8] Sneed characterized theoretical terms relativized to the theory in question, i.e. a term *t* is theoretical with respect to a theory *T* or *T-theoretical*:

Definition 1 (T-theoreticity)
A term *t* is theoretical with respect to the theory *T*, or for short, *T*-theoretical if and only if any method of determining the extension of *t*, or some part of that extension, rests on some axiom of *T*.

Then, Sneed also defined what is *T-non-theoreticity*:

Definition 2 (T-non-theoreticity)
A term *t* is T-non-theoretical if and only if it is not *T*-theoretical.

If we remove the *T*-theoretical terms in the definition of *T*'s potential models, this leads to structures that are to be treated on a purely empirical layer; we call the class $M_{pp}(T)$ of these structures of theory *T* its "partial potential models".

Finally, every physical theory has a class *I* of intended systems (or applications) and, of course, different intended systems of a theory may partially overlap. This means that there is a class *C* of constraints that produces cross connections between the overlapping intended systems. In brief, this structuralist view of scientific theories regards the core *K* of a theory as a quadruple $K = \langle M_p, M_{pp}, M, C\rangle$. This core can be supple-

[2] Lakatos fled to Austria (1956) and later to England.

mented by the class I of intended applications of the theory $T = \langle K, I \rangle$. To make it clear that this concept reflects both sides of scientific theories, these classes of K and I are shown in fig. 1. Thus we notice that M_{pp} and I are entities of an empirical layer, whereas M_p and M_{pp} are structures in a theoretical layer of the schema.

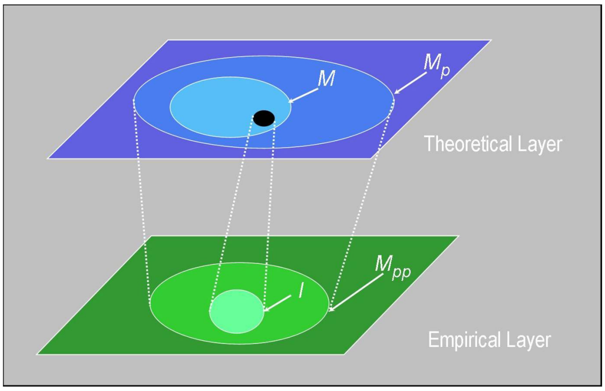

Fig. 1. Empirical and theoretical structural layers..

3. An early view on fuzzy structuralism

In the last two decades of the 20th century we find a development to apply Metastructuralism also in philosophy of medicine that also had a link to the theory of Fuzzy sets: Since the 1980 the philosopher of medicine and physician Kazem Sadegh-Zadeh (born 1942) has been discussing the meaning of concepts in medicine [20-23]. Because "health", "illness" and "disease" are notions originated in the theory of medicine that can't be defined in classical logic and because "health is a matter of degree, illness is a matter of degree, and disease is a matter of degree" [23] he fuzzified these concepts. In 1982, within the framework of a conference of medicine and philosophy, he blurred the notion "patienthood" (being afflicted by a malady) as a new notion in the theory of medicine "of which the notion of health will be a fuzzy additive inverse" in the sense [20]:[3]

$$\text{health} = 1 - \text{patienthood}.$$

With congratulations to Lotfi A. Zadeh's 80th birthday Sadeh-Zadeh published in the year 2001 his article "The Fuzzy Revolution: Goodbye to the Aristotelian Weltanschauung" [26]. Referring to Wolfgang Stegmüller's *The Structure and Dynamics of Theories* [3] he stated in this article that the concepts of Popper, Kuhn and their combatants "are still too vague and inadequate to be useful, [...] we may, nevertheless, learn from these studies that in contrast to our accustomed views on the development of science and scientific knowledge, this very development is not a cumulative process. Science does not progress continuously and by accumulating knowledge. It does not add to an antecedent knowledge or theory T_i a subsequent knowledge or theory T_{i+1} of the same type such that one could reasonably consider science as the open ordered series of related theories T_1, T_2, ..., T_{i+1}. Scientific ideas, theories, and worldviews evolve discontinuously in that a body of knowledge or theory T_i, which is held over a particular period of time, is dislodged by another body of knowledge or theory T_j, because the disciplinary matrix within which the former theory T_i had grown, changes to another disciplinary matrix which gives rise to the new theory, T_j, that is incompatible and incommensurable with its predecessor T_i." [24]

In his *Handbook on Analytical Philosophy of Medicine* that just appeared this year [27], Sadegh-Zadeh demanded an "overhaul" to adapt the structuralist metatheory to fuzzy set theory ([28], p. 439f). Then he required: "To render the metatheory applicable to real world scientific theories, it needs to be fuzzified because like everything else in science, scientific theories are vague entities and implicitly or explicitly fuzzy." He then lists two ways of scientific theories' explicit fuzzifications:

a) Introduction of the theory's set-theoretical predicate as a fuzzy predicate ("x is a fuzzy S" instead of "x is an S").

b) In addition to a) also any other component of the theory appearing in the structure that defines the predicate may be fuzzified.

Unfortunately, Sadegh-Zadeh goes not into details at this point but he concludes this section with an outlook: "Fuzzifications of both types will impact the application and applicability of theories as well as the nature of the knowledge produced by using them. This is true because fuzzification will change the conception of models; potential models; partial, potential models; and the core and intended applications of a theory, on the one hand; and the epistemological relationships between empirical claims of the theory and the 'real world', on the other, e.g., support, confirmation, falsification, etc." ([27], p. 441)

At the end of his chapter "The Architecture of Medical Knowledge" Sadegh-Zadeh writes: "The above considerations suggest that the entities a theory is concerned with, be construed as vague entities." For similar analyses and assessments he referred to my papers [29-31]. Therefore we will turn now to these ideas on a fuzzy structuralist view of scientific theories in general.

4. Fuzzy metastructuralism

The fuzzification of scientific theories in the metastructuralist approach the proposed modification of this approach pertains to the real layer in fig. 1. A distinction can be made between real systems and phenomena, on the one hand, and perceptions of these entities, on the other. Thus a lower layer – the real layer – is introduced and the former empirical layer is renamed the "fuzzy layer", as the partial potential models and intended systems are not real systems because a minimal structure is imposed by the scientist's observations. These are perception-based systems and thus must be distinguished

[3] For a review of Sadegh-Zadeh's work in the fields of Fuzzy Sets and Philosophy of Medicine see [24].

from real systems and phenomena that have no structure before someone imposes one upon them.

Now there is a layer of perceptions between the layer of real systems and phenomena and the layer of theoretical structures. In accordance with Zadeh's computational theory of perceptions (CTP), perceptions in this intermediate layer can be represented as fuzzy sets. Whereas measurements are crisp, perceptions are fuzzy, and because of the resolutions achieved by our sense organs (e.g. aligning discrimination of the eye), perceptions are also granular – in 2001 Zadeh wrote in the *AI Magazine*: "perceptions, in general, are both fuzzy and granular or, for short *f*-granular [32]. Fig. 2 shows Zadeh's depiction of *crisp* (*C*) and *fuzzy* (*F*) granulation of a linguistic variable.

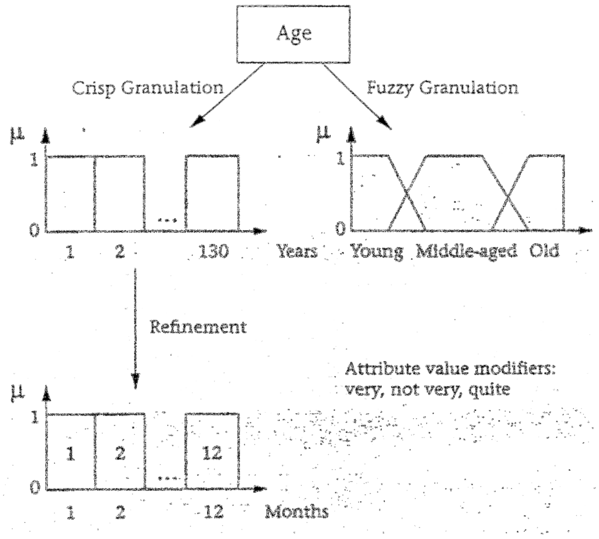

Fig. 2. *F*-Granulation and *C*-Granulation of *Age* [32].

When Zadeh established CTP on the basis of computing with words (CW), which in turn is based on his theory of fuzzy sets and systems [33, 34], he earnestly believed that these methodologies would attain a certain importance in science: "In coming years, computing with words and perceptions is likely to emerge as an important direction in science and technology." [35]. Taking Zadeh at his word, we incorporate the methodologies of fuzzy sets, fuzzy relations, and computing with words and perceptions into our metastructuralist approach with intent to obtain fuzzy structures of scientific theories.

As discussed above, a fuzzy layer of perceptions is between the real layer (real systems and phenomena), and the theoretical layer (structures of models and potential models) (Fig. 3). Thus the relationship between real systems and theoretical structures is now split in two parts: fuzzification and defuzzification.

4.1. Fuzzification

Measurements are crisp and perceptions are fuzzy and granular. To represent perceptions we can use fuzzy sets, e.g. A^F, B^F, C^F, ... It is also possible that a scientist observes not just a single phenomenon, but interlinked phenomena, e.g. two entities move similarly or inversely, or something is faster or slower than a second entity, or is brighter or darker, or has an analogous smell, etc. Such relationships can be characterized by *fuzzy-relations* f^F, g^F, h^F, ….

4.2. Defuzzification

"Measure what is measurable and make measurable what is not so" is a sentence attributed to Galileo. In modern scientific theories this is the way to get from perceptions to measurements or quantities to be measured. Here this transfer is interpreted as a defuzzification from perceptions represented by fuzzy sets A^F, B^F, C^F, … and relations between perceptions represented by fuzzy relations f^F, g^F, h^F, … to ordinary (crisp) sets A^C, B^C, C^C, … and relations f^C, g^C, h^C, … These sets and relations are basic entities for the construction of (potential) models of a scientific theory in the theoretical layer.

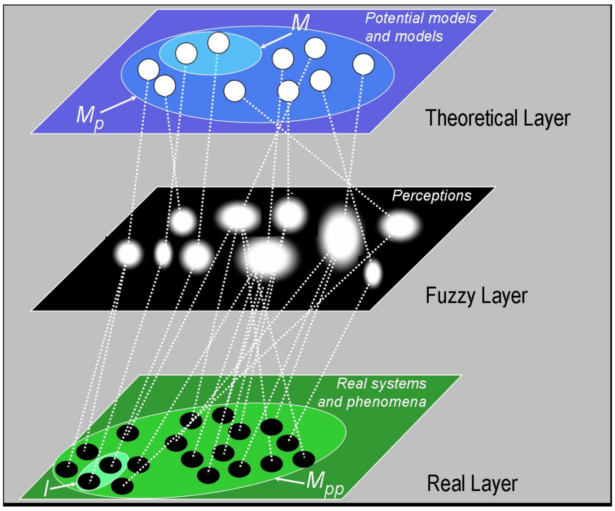

Fig. 3. Empirical, fuzzy, and theoretical layers of crisp and fuzzy structures in scientific research.

4.3. Initial theoretization

From real systems to theoretical structures we model in two steps: the first is fuzzification from real systems to representations of our perceptions and the second is from these fuzzy sets and fuzzy relations to potential models of a scientific theory by defuzzification Therefore, we can say that the serial operation of fuzzification and defuzzification yields to an "initial theoretization", because this path initially gives real phenomena and systems a theoretical meaning. We use the name "initial" because it starts from a non-theoretical layer i.e. there was no scientific theory but only pre-theoretical phenomena.

5. Theoretization of scientific theories

When the metastructuralist relation "Theoretization" is not an initial theoretization (i.e. starting from pre-theoretical phenomena) the relation connects a layer of (potential) models of, say, theory *T*, with a layer of (potential) models of theory *T'*. As we mentioned in Section 2.3, in this view all theoretical terms are theoretical relative to a theory, i.e. a concept is not theoretical at all but it is *T*-theoretical or *T'*-theoretical to the respective theory *T or T'*.

A theoretization between theories *T'* and *T* exists if *T'* results from *T* by adding new theoretical terms and introduction of new laws that connect the former theoretical terms of theory *T* with this new theoretical terms in theory *T'*. On the other hand, if we remove all *T*-theoretical terms of a theory *T* in its potential models $M_p(T)$, then the remaining structures can be viewed as structures in a *T*-non-theoretical layer that we call "partial potential models" of theory *T* and we name their set $M_{pp}(T)$.

Also we said there that every empirical theory *T* has a class *I* of intended application systems that is a subset of all partial potential models in $M_{pp}(T)$. These meta-structuralist concepts are shown in Fig. 4: The sets $M_p(T)$ and $M(T)$ and the sets $M_{pp}(T)$ and *I* are located in different "layers". The latter two items are structures in the *T*-non-theoretical layer, whereas $M_p(T)$ and $M(T)$ are structures in a theoretical layer of this schema. The spotted lines indicate the relation between the two layers that shows theory *T'* as a "Theoretization" of theory *T*. However, this "Theoretization" is a set-theoretical relation for it holds: *T'* is a theoretization of *T* if and only if $M_{pp}(T) \subseteq M(T)$.

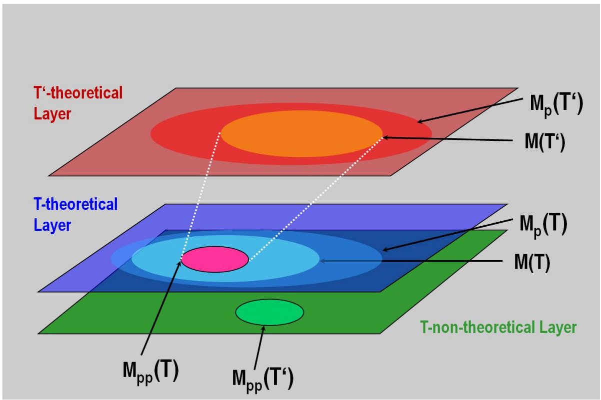

Fig. 4. The theoretical structures of a theory *T*, its theoretical structures, the structures of its Theoretization *T'* and the *T*-non-theoretical structures.

Thus, for the intertheoretic relation of theoretization we have the following properties:

- The new theory *T'* adds a new theoretical layer to the old theory *T*.
- *T*-theoretical terms are not *T'*-theoretical but *T'*-non-theoretical terms, and reciprocally *T'* –theoretical terms may not be any of the *T*-non-theoretical terms.
- The old theory must not be changed in any way by the new theory.

In this manner the space-time theory arose from Euclidean geometry when the term "time" was added to the term "length", and classical kinematics developed from classical space-time theory when the term "velocity" was added. Classical kinematics turned into classical (Newtonian) mechanics when the terms "force" and "mass" were introduced.

Furthermore, successive adding of new theoretical terms to a theory establishes a hierarchy of theories and a comparative concept of theoreticity. In this hierarchy it holds that the higher in the hierarchy the more theoretical terms exist and the lower layers are characterized by the non-theoretical basic of the theory.

What happens in the lowest layer of this hierarchy? – Here exists a theory *T* with theoretical terms and relations but it is not a theoretization of another theory. This theory *T* covers phenomena and intended systems initially with theoretical terms. This is again what we named an "initial theoretization" because here the *T*-theoretical terms are the only theoretical terms of this structure. They have been derived directly as measurements from observed phenomena. In our fuzzy-structuralist view, this initial theoretization is a serial connection of fuzzification and defuzzification.

6. Theoretization and empirization

Discussing my talk at the *IFSA 2007 World Congress* in Cancun, Mexico, in June 2007 [29] it was Jerry Mendel's suggestion to substitute the fuzzy layer between the real and the theoretical layers by the whole space between these two layers as the "space" of fuzzy entities. – Fig. 5 shows my actualization of this idea by a "Fuzzy Space" of perceptions between the theoretical and the empirical layer.

We introduce the variable *T* – "Theoretization" – which can be interpreted as membership function of perceptions in the class of theoretical entities ((potential) models). A perceptions *p* with $T(p) = 1$ is completely theoretical and if $T(p) = 0$ then perception *p* is completely empirical.

We also introduce the variable *E* – "Empirization" – which is the complement of the theoretization *T*. A perception *p* with $E(p) = 1$ is completely empirical and $E(p) = 0$ means that *p* is completely theoretical. Therefore we have got empirization as the complement of theoretization:

$$E = 1 - T.$$

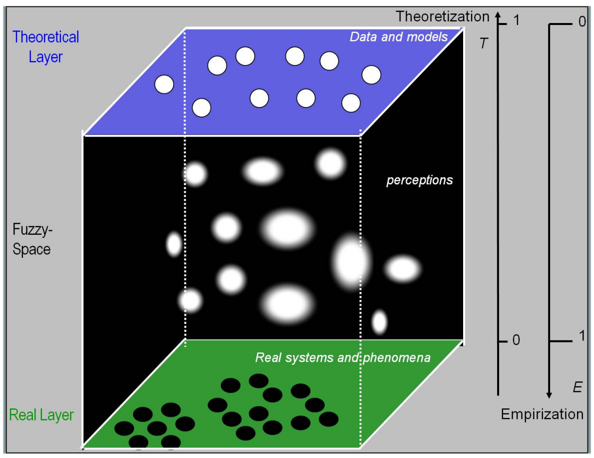

Fig. 5. The fuzzy space of perceptions between the empirical and the theoretical layer. "Theorization" and "Empirization".

7. Reduction of scientific theories

Metastructuralists try to reconstruct the change of theories, the so-called “scientific revolutions” or paradigm shifts, e.g. the change from Ptolemy’s geocentric universe to Copernicus’ heliocentric world picture or from Newtonian Mechanics to Einstein’s Special Relativity Theory by the another intertheoretical relation that is called “Approximate Reduction”. However, this “Approximate Reduction” is – as its name says, just an approximation of the “pure” intertheoretical relation of “Reduction” and in a future paper we will interpret the “approximation” as a fuzzification! Here, we give just the definition of the classical “Reduction”-relation and Fig. 6 shows how it works.

7.1. Definition (reduction)

There are two theories, say T_{old} and T_{new}. T_{old} reduces T_{new} by the reduction relation ρ the following conditions are fulfilled:

1. $\rho \subseteq M_p(T_{old}) \times M_p(T_{new})$
2. $\forall x,x'$: if $<x,x'> \in \rho$ and $x' \in (T_{new})$, then $x' \in M(T_{old})$.

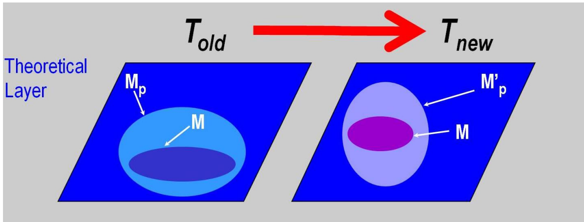

Fig. 6. The relation “Reduction” between theories T_{old} and T_{new}.

7.2. “Fuzzy reduction”

The intertheoretical relation “Reduction” cannot be the best choice to represent theory changes as in scientific revolutions because these paradigm shifts are not pure rational changes and between the old and the new theory there is no one-to-one-relation. Therefore we have to respect some fuzziness in these transformations that cannot be represented by hard mathematics! Nevertheless, metastructuralists proposed methods to reconstruct the “Approximative Reduction” by methods of classical mathematics, e.g. converging series of models of a theory or topological entities in spaces of such models. In a future paper we will start working to define an approximative version of the intertheoretical relation of Reduction by using fuzzy methods. This “Fuzzy Reduction” will be the most interesting intertheoretical relation in Fuzzy Structuralism because it could act as an appropriate modeling for paradigm changes in history of science.

8. Conclusions

In this paper we used Zadeh’s Fuzzy Set Theory and his Computational Theory of Perceptions (CTP) as an appropriate methodology to represent efforts of scientific research to bridge the gap between real phenomena and systems, empirical observations and the abstract construction of theoretical structures. In the classical structuralist view of theories (Metastructuralism) there is a real layer of real phenomena and systems and a theoretical layer of potential models and models that are structured entities. But there is no representation of the observer’s role and his/her perceptions.

The modified view of the structuralist approach or Metastructuralism presented in this paper is a proposal that will be worked out in detail under the names of “Fuzzy structuralist view on theories” and “Fuzzy-Metastructuralism” in the next future. This new approach comprises a layer of fuzzy sets and fuzzy relations as a means of dealing with the difference between real phenomena and systems on the one hand and the observer’s perceptions of these real entities on the other. This extended view of scientific theories may open up a new and fruitful way to understand scientific research.

The Fuzzy structuralist view on theories uses fuzzy sets and fuzzy relations to represent perceptions as important components in the interpretation of scientific theories. This is very suitable, because in new physical theories of the 20th century, such as relativity theory and quantum mechanics, the observer and his/her perceptions play a central role, and this is also the case in evolutionary biology and medical diagnostics. [29-31]

This work will be continued and finally this should breed to a new concept in philosophy of science.

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