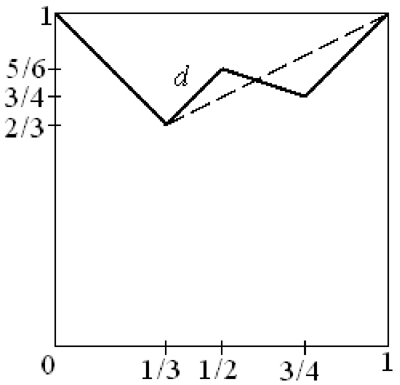

Figure 1: The dependence function d from Example 2 that is not convex at point 1.

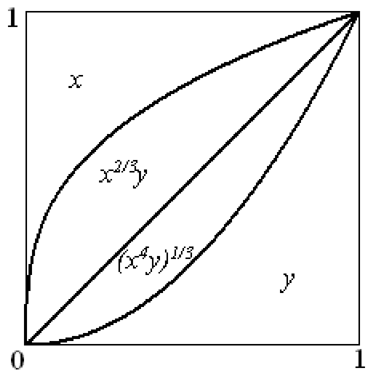

Figure 2: The aggregation function A generated by the dependence function d from Example 2.

see Fig. 2.
A is 1-Lipschitz in the second variable, i.e., $A(x, .)$ is 1-Lipschitz for each x, but A is not 1-Lipschitz in the first variable. Indeed, if we choose the points $\mathbf{u} = (0.9, 0.81)$ and $\mathbf{v} = (0.81, 0.81)$, then $A(0.9, 0.81) = 0.81$, $A(0.81, 0.81) = \sqrt[3]{0.9^{10}} = 0.703842$, and thus,

$$\begin{aligned} A(0.9, 0.81) - A(0.81, 0.81) &= 0.106158 \\ &> 0.09 = 0.9 - 0.81, \end{aligned}$$

which confirms that A is not 1-Lipschitz in the first variable.

4. Concluding remarks

We have clarified the structure of the class of 1-Lipschitz power stable aggregation functions, including the prominent subclass of power stable quasi-copulas. As the main result we have shown the relationship of power stable quasi-copulas and dependence functions convex at points 0 and 1, satisfying the property $d(0) = d(1) = 1$. For more details and some other related results see [8]. In our further investigation, we open the problem of characterization of 1-Lipschitz power stable aggregation functions of higher dimensions.

Acknowledgement The authors kindly acknowledge the support of the grant VEGA 1/0419/13 and the project of Science and Technology Assistance Agency under the contract No. APVV-0073-10. Moreover, the work of the second author on this paper has been done in connection with project IT4Innovations Centre of Excellence, reg. no. CZ.1.05/1.1.00/ 02.0070 supported by Research and Development for Innovations Operational Programme financed by Structural Founds of Europe Union and from the means of state budget of the Czech Republic.

References

[1] C. Alsina, R.B. Nelsen, B. Schweizer, On the characterization of a class of binary operations on distributions functions. *Stat. Probab. Lett.* **17** (1993) 85-89.

[2] J. Beirlant, Y. Goegebeur, J. Segers, and J. Teugels, *Statistics of Extremes: Theory and Applications.* John Wiley & Sons Ltd., Chichester, 2004.

[3] G. Beliakov, A. Pradera, and T. Calvo, *Aggregation Functions: A Guide for Practitioners.* Springer, Heidelberg, Berlin, New York, 2007.

[4] C. Genest, J.J. Quesada Molina, J.A. Rodríguez-Lallena, and C. Sempi, A characterization of quasi-copulas. *J. Multivariate Anal.* **69** (1999) 193-205.

[5] M. Grabisch, J.-L. Marichal, R. Mesiar, and E. Pap, *Aggregation Functions.* Cambridge University Press, Cambridge, 2009.

[6] G. Gudendorf, J. Segers, Extreme-Value copulas. In *Proc. of Workshop on Copulas Theory and its Applications.* F. Durante et al., eds., 2010

[7] A. Kolesárová, R. Mesiar, and T. Rückschlossová, Power stable aggregation functions. *Fuzzy Sets and Systems*, doi. 10.1016/j.fss.2013.05.005.

[8] A. Kolesárová, R. Mesiar, 1-Lipschitz power stable aggregation functions. *Inform. Sci.*, submitted.

[9] R.B. Nelsen, *An Introduction to Copulas.* Vol. 139 of *Lecture Notes in Statistics*, Springer, New York, second edition, 2006.

[10] A. Sklar, Fonctions de répartition à n dimensions et leurs marges. *Publ. Inst. Statist. Univ. Paris* **8** (1959) 229-231.