**Proposition 8** *We have*

$$I_{i,j}(U) = I_{i,j}(U_{i,j})$$

*where*

$$I_{i,j}(U_{i,j}) = \frac{1}{(a_i^{p_i} - a_i^1)(a_j^{p_j} - a_j^1)} \Big[\widehat{u}_{i,j}(a_i^1, a_j^1) - \widehat{u}_{i,j}(a_i^1, a_j^{p_j}) - \widehat{u}_{i,j}(a_i^{p_i}, a_j^1) + \widehat{u}_{i,j}(a_i^{p_i}, a_j^{p_j})\Big]$$

## 8. Conclusion

This paper is devoted to the use of the GAI model in a MCDM context.

Firstly, the problem of which conditions are relevant to add on the GAI model to make it more easy to learn and interpret is addressed. From a user point of view, the partial utility functions $u_S$ appearing in a GAI model are more easily interpreted if there are non-negative and monotonic. We consider a particular case of 2-additive GAI, where the definition is borrowed from the Choquet integral setting. In particular, we show that it is sufficient to consider only positive terms in the expression of a 2-additive GAI model.

Secondly, we are interested in the learning phase. As in references [4, 5], we interpret the learning examples provided by the user as constraints, which yields the use of linear programming to learn a GAI model. Our contribution is on handling inconsistent learning examples. We first generate several possible repairs (set of learning examples that need to be changed or removed to recover consistency). We also provide a new approach for explaining an inconsistency, based on the Farkas lemma.

Finally, we propose some indices to interpret a GAI model, based on a generalization of importance and interaction indices already at work for the Choquet integral. The give the expression of these indices for 2-additive GAI models.

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