Remark 5.8. From (15) and (11) we obtain

$$V_3(A) = V_1(A) + \frac{(s-t)(d_2-d_1)}{d_1+d_2}(1-\sigma_2)\,. \quad (16)$$

Equation (16) shows that the weakness of $V_1(A)$ described in Remark 4.5 does not happen using the evaluation $V_3(A)$.
Moreover, from (15) and (12) we get

$$V_3(A) = V_2(A)\sigma_2 + \frac{td_1+sd_2}{d_1+d_2}(1-\sigma_2)\,. \quad (17)$$

Equation (17) shows that the weakness of $V_2(A)$ described in Remark 4.6 does not occur for the evaluation $V_3(A)$.

6. Conclusion

Following the words of Grzegorzewski in [9] that, in his introduction, underlines the importance to develop interval approximation for general fuzzy sets, in this paper we introduce, for the first time, the interval approximation of a fuzzy quantity. Working with fuzzy numbers, this type of operation means to find the interval nearest to the original fuzzy number respect some type of metric. In fuzzy quantities' case, that are the typical outputs of a fuzzy control system, the introduction of a metrics is not so trivial, so in this first paper we have spoken of the interval nearest to the original fuzzy quantity respect a general functional. The functional we use is suggested by the distance proposed by Bertoluzza et al. [3] and generalized by Trutshnig et al. [11]. This distance depends by a parameter that modifies the weight of the "spread" part, but the nearest interval founded doesn't depends by it. Even in the formulation for fuzzy quantities this happens so our following study will be in the direction to understand why this fact happens and how to modify this situation. Another direction is to modify the functional we start and to try to find a sort of functional that should be a distance. As we have use this approach so as to evaluate a fuzzy quantity, having in mind a defuzzification problem, we have compared our results with other previous methods introduced by other authors finding a unifying view, we have left behind the method proposed by Facchinetti and Pacchiarotti [5]. This happens as their formulation is a geometrical view of Fortemps and Roubens idea in a more general case. Their proposal differs for two reasons. They suppose that the midpoint is not an imposed choice, but that it is possible to select a point of the interval depending by optimistic or pessimistic attitude of decision maker and that the measure that appear in the evaluation is not necessarily a Lebesgue measure but may be more general. Even in this direction we will try to find a unifying view.

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