Optimum allocation of centers in transportation networks by means of fuzzy graph bases

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Abstract

In this paper the questions of defining the optimum allocation of centers in fuzzy transportation networks are observed by the minimax criterion. It is supposed that the information received from the geographical information system is presented as a fuzzy graph. In this case the task of defining optimum allocation of the centers transforms into the task of defining the fuzzy set of graph bases. The example of finding the optimum allocation of centers for railway stations GIS is considered.

Keywords: Fuzzy graph, fuzzy directed way, accessible degree, fuzzy transitive closure, fuzzy base, fuzzy set of bases

1. Introduction

The large-scale increasing and versatile introduction of a geographical information system (GIS) is substantially connected with the necessity of the perfection of information systems providing decision-making. GIS are applied practically in all spheres of human activity. Geographical information technologies have now reached an unprecedented position, offering a wide range of very powerful functions such as information retrieval and display, analytical tools, and decision support [1, 2]. Unfortunately, geographical data are often analyzed and communicated through largely non-negligible uncertainty. Uncertainty exists in the whole process from geographical abstraction, data acquisition, and geoprocessing to the use [3, 4].

One of the tasks solved with GIS is the task of the centers allocation [5]. The search of the optimum placing of hospitals, police stations, fire brigades and many other necessary important enterprises and services on some sites of the considered territory are confined to this task. In some cases the criterion of optimality can consist in the minimization of the journey time (or in the minimization of distances) from the service centre to the most remote service station. In other cases the criterion of optimality consists in the choice of such a place of allocating the centres so that the route from them to any other place of service could be passed best by some criterion of a way. In other words, the problem is the optimization of "the worst variant» [6].

However, very often, the information represented in GIS, happens to be of the approximate value or insufficiently authentic [7]. We consider that a certain railway system has n railway stations. There are k service centres, which may be placed into these railway stations. Each centre serves a station, and also some neighbouring stations with the given degree of service. The centres can fail during the exploitation. It is necessary for the given number of centres to define their best allocation. In other words, it is necessary to define the places of k centres into n railway stations so that the "control" of all the territory (all the railway stations) is carried out with the greatest possible service degree.

2. Basic concepts and definitions

It is supposed that the service degree of region is defined as the minimal value from the service degrees of each area.

Taking into account that the service degree cannot always have symmetry property (for example, by specific character and relief of the region) the model of such task is a fuzzy directed graph $\tilde{G}=(X,\tilde{U})$ [8]. Here, set $X=\{x_i\}$, $i \in I=\{1,2,\dots,n\}$ is a set of vertices and $\tilde{U}=\{<\mu_U<x_i,x_j>/<x_i,x_j>>\}$, $<x_i,x_j>\in X^2$ is a fuzzy set of directed edges with a membership function μ_U:$X^2\to[0,1]$. The membership function $\mu_U<x_i,x_j>$ of fuzzy graph $\tilde{G}=(X,\tilde{U})$ defines a service degree of area j in the case when the center is placed into area i. We assume that the service degree has the property of transitivity, i.e. if the service centre is in area x_i and serves area x_j with degree $\mu_U<x_i,x_j>$, and if the service centre is in area x_j and serves area x_k with degree $\mu_U<x_j,x_k>$ then service degree of area x_k from area x_i is $\mu_U<x_i,x_j>\&\mu_U<x_j,x_k>$. Here operation & is minimum operation.

The task of the best allocation of centres on the fuzzy graph may be limited to the problem of finding a subset of vertices B, from which all the other vertices of the fuzzy graph X/B achievable with the greatest degree. For considering the questions of the optimum allocation of the service centres we shall focus on the concepts of a fuzzy directed way, and the fuzzy base of a fuzzy graph [8, 9].

Fuzzy directed way $\tilde{L}(x_i,x_j)$ of graph $\tilde{G}=(X,\tilde{U})$ is called the sequence of fuzzy directed edges from vertex x_i to vertex x_m:

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$\widetilde{L}(x_i, x_m) = <\mu_U < x_i, x_j > / < x_i, x_j >>, < \mu_U < x_j, x_k > / < x_j, x_k >>, \ldots, < \mu_U < x_l, x_m > / < x_l, x_m >>.$

The conjunctive durability of way $\mu(\widetilde{L}(x_i, x_j))$ is defined as

$$\mu\,(\widetilde{L}(x_i,x_m)) = \underset{<x_\alpha,x_\beta>\in\widetilde{L}(x_i,x_m)}{\&} \mu_U < x_\alpha, x_\beta >.$$

The fuzzy directed way $\widetilde{L}(x_i, x_j)$ is called a simple way between vertices x_i and x_m if its part is not a way between the same vertices.

Vertex y is called the fuzzy accessible of vertex x in graph $\widetilde{G} = (X, \widetilde{U})$ if a fuzzy directed way from vertex x to vertex y exists.

The accessible degree of vertex y from vertex x, $(x \neq y)$ is defined by the following expression

$$\gamma(x,y) = \max_{\alpha}(\mu(\widetilde{L}_\alpha(x,y))), \alpha=1,2,\ldots,p,$$

where p is the number of various simple directed ways from vertex x to vertex y.

On the basis of the presented definition of a fuzzy accessible vertex we can construct an accessible matrix N, containing accessible degrees for all pairs of vertices

$$N = \|\gamma_{ij}\|_n = \bigcup_{k=0}^{n-1} R^m.$$

Here, $\gamma_{ij}=\gamma(x_i,x_j)$, $x_i x_j \in X$, R^m is the vertex matrix power m for a graph (matrix R^0 is an individual matrix).

Let us consider that each vertex $x \in X$ in the graph $\widetilde{G}=(X,\widetilde{U})$ is accessible from itself with the accessible degree $\gamma(x,x)=1$.

Example 1. For the fuzzy graph 1 presented on Fig.1, vertex x_5 is a fuzzy accessible vertex from x_1 with the accessible degree

$$\gamma(x_1, x_5) = \max\{(0{,}7\,\&\,0{,}3);(0{,}6\,\&\,0{,}8)\} =$$
$$= \max\{0{,}3;0{,}6\} = 0{,}6.$$

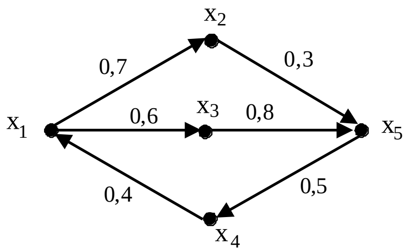

Fig. 1: Fuzzy graph.

Let the fuzzy graph $\widetilde{G} = (X, \widetilde{U})$ be given. The fuzzy multiple-valued reflections $\widetilde{\Gamma}^1$, $\widetilde{\Gamma}^2$, $\widetilde{\Gamma}^3$,..., $\widetilde{\Gamma}^k$ are defined as:

$\widetilde{\Gamma}^1(x_i) = \{< \mu_{\Gamma^1(x_i)}(x_j)/(x_j) >\}$, where

$(\forall x_j \in X)[\mu_{\Gamma^1(x_i)}(x_j) = \mu_U < x_i, x_j >]$,

$\widetilde{\Gamma}^2(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}(x_i)\}$, $\widetilde{\Gamma}^3(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}^2(x_i)\}$, ...,

$\widetilde{\Gamma}^k(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}^{k\text{-}1}(x_i)\} = \{< \mu_{\Gamma^k(x_i)}(x_j)/x_j >\}$, where

$(\forall x_j \in X)[\mu_{\Gamma^k(x_i)}(x_j) = \underset{\forall x_l \in X}{\vee} \mu_{\Gamma^{k-1}(x_i)}(x_l)\,\&\,\mu_U < x_l, x_j >]$.

It is obvious that $\widetilde{\Gamma}^k(x_i)$ is a fuzzy subset of vertices, which are accessible from vertex x_i, using fuzzy ways of length k.

Example 2. For the fuzzy graph presented in Fig.1 we have

$\widetilde{\Gamma}^1(x_1) = \{< 0{,}7/(x_2)>, < 0{,}6/x_3 >\}$,

$\widetilde{\Gamma}^2(x_1) = \{< 0{,}6/x_5 >\}$.

The fuzzy transitive closure $\hat{\Gamma}(x_i)$ is the fuzzy multiple-valued reflection:

$$\hat{\Gamma}(x_i) = \widetilde{\Gamma}^0(x_i) \cup \widetilde{\Gamma}(x_i) \cup \widetilde{\Gamma}^2(x_i) \cup \ldots = \bigcup_{j=0}^{\infty} \widetilde{\Gamma}^j(x_i).$$

Where $\widetilde{\Gamma}^0(x_i) = \{<1/x_i>\}$. In other words, the vertex x_i can be reached herself from herself with degree 1.

The set $\hat{\Gamma}(x_i)$ is the fuzzy subset of vertices, which are accessible from vertex x_i by some fuzzy way with the greatest possible conjunctive degree. As we consider the final graphs, it is possible to state, that

$$\hat{\Gamma}(x_i) = \bigcup_{j=0}^{n-1} \widetilde{\Gamma}^j(x_i).$$

Example 3. For the fuzzy graph presented in Fig.1, the fuzzy transitive closure of vertex x_1 is:

$\hat{\Gamma}(x_1) = \{<1/x_1>, <0{,}7/x_2>, <0{,}6/x_3>, <0{,}5/x_4>, <0{,}6/x_5>\}$.

Let's define fuzzy reciprocal multiple-valued reflections $\widetilde{\Gamma}^{-1}$, $\widetilde{\Gamma}^{-2}$, $\widetilde{\Gamma}^{-3}$,..., $\widetilde{\Gamma}^{-k}$ as:

$\widetilde{\Gamma}^{-1}(x_i) = \{< \mu_{\Gamma^{-1}(x_i)}(x_j)/(x_j) >\}$, where

$(\forall x_j \in X)[\mu_{\Gamma^{-1}(x_i)}(x_j) = \mu_U < x_j, x_i >]$,

$\widetilde{\Gamma}^{-1}(x_i) = \widetilde{\Gamma}^{-1}\{\widetilde{\Gamma}^{-1}(x_i)\}$, $\widetilde{\Gamma}^{-3}(x_i) = \widetilde{\Gamma}^{-1}\{\widetilde{\Gamma}^{-2}(x_i)\}$, ...,

$\widetilde{\Gamma}^{-k}(x_i) = \widetilde{\Gamma}^{-1}\{\widetilde{\Gamma}^{-(k-1)}(x_i)\} = \{< \mu_{\Gamma^{-k}(x_i)}(x_j)/x_j >\}$,

where

$(\forall x_j \in X)[\mu_{\Gamma^{-k}(x_i)}(x_j) = \underset{\forall x_l \in X}{\vee} \mu_{\Gamma^{k-1}(x_i)}(x_l)\,\&\,\mu_U < x_j, x_l >]$.

It is obvious, that $\widetilde{\Gamma}^{-k}(x_i)$ is a fuzzy subset of vertices, from which is accessible to reach vertex x_i, using fuzzy ways of length k.

Example 4. For the fuzzy graph presented in Fig.1, we have

$\widetilde{\Gamma}^{-1}(x_1) = \{< 0{,}4/x_4 >\}$, $\widetilde{\Gamma}^{-2}(x_1) = \{< 0{,}4/x_5 >\}$.

The fuzzy reciprocal transitive closure $\hat{\Gamma}^{-}(x_i)$ is the fuzzy reciprocal multiple-valued reflection

$$\hat{\Gamma}^{-}(x_i) = \widetilde{\Gamma}^0(x_i) \cup \widetilde{\Gamma}^{-1}(x_i) \cup \widetilde{\Gamma}^{-2}(x_i) \cup \ldots = \bigcup_{j=0}^{n-1} \widetilde{\Gamma}^{-j}(x_i).$$

In other words, $\hat{\Gamma}^{-}(x_i)$ is a fuzzy subset of vertices, from where vertex x_i can be reached by some fuzzy way with the greatest conjunctive degree.

Example 5. For the fuzzy graph presented in Fig.1, the fuzzy reciprocal transitive closure of vertex x_1 is

$\hat{\Gamma}^{-}(x_1) = \{<1/x_1>, <0{,}3/x_2>, <0{,}4/x_3>, <0{,}4/x_4>, <0{,}4/x_5>\}$.

Fuzzy graph $\tilde{G} = (X, \tilde{U})$ is named a fuzzy strongly connected graph if condition

$$(\forall x_i \in X)(S_{\hat{\Gamma}(x_i)} = X). \qquad (1)$$

is satisfied.

Here $S_{\hat{\Gamma}(x_i)}$ is a carrier of fuzzy transitive closure $\hat{\Gamma}(x_i)$.

In contrast, fuzzy graph $\tilde{G} = (X, \tilde{U})$ is a fuzzy strongly connected graph if there is a fuzzy directed way with the conjunctive durability distinct from 0 between any two vertices.

It is easy to show, that expression (1) is equivalent to expression (2)

$$(\forall x_i \in X)(S_{\hat{\Gamma}^{-}(x_i)} = X). \qquad (2)$$

Here $S_{\hat{\Gamma}^{-}(x_i)}$ is the carrier of the fuzzy reciprocal transitive closure $\hat{\Gamma}^{-}(x_i)$.

Let fuzzy transitive closure for vertex x_i look like $\hat{\Gamma}(x_i) = \{<\mu_{i1}(x_1)/x_1>, <\mu_{i2}(x_2)/x_2>, ..., <\mu_{in}(x_n)/x_n>\}$, then the volume $\rho(\tilde{G}) = \underset{j=\overline{1,n}}{\&}\ \underset{i=\overline{1,n}}{\&} \mu_{ij}(x_j)$ is termed as the degree of the fuzzy graph strong connectivity.

Let $\tilde{G} = (X, \tilde{U})$ be a fuzzy graph with degree of strong connectivity $\rho(\tilde{G})$, and $\tilde{G}' = (X', \tilde{U}')$ be a fuzzy subgraph with the degree of strong connectivity $\rho(\tilde{G}')$.

Fuzzy subgraph $\tilde{G}' = (X', \tilde{U}')$ is referred to as a maximum strong connectivity fuzzy subgraph or a fuzzy strong connectivity component if there is no other subgraph $\tilde{G}'' = (X'', \tilde{U}'')$ for which $\tilde{G}' \subset \tilde{G}''$, and $\rho(\tilde{G}') \le \rho(\tilde{G}'')$.

Definition 1. A fuzzy base with the degree $\alpha \in [0,1]$ is called a subset vertices $B_\alpha \subset X$ from which any vertex of a fuzzy graph is accessible with the degree not less than α and which is minimal in the sense that there is no subset $B' \subset B_\alpha$, having the same accessible property.

Formally, any subset of the vertices of the fuzzy graph can be considered as a fuzzy base with an α degree.

Let's designate through $\tilde{R}(B)$ a fuzzy set of vertices, accessible of any subset $B \subset X$. Then set B_α is a fuzzy base with degree α only in case the following conditions are carried out

$$\tilde{R}(B_\alpha) = \{<\mu_j/x_j> \mid x_j \in X \& (\forall j = \overline{1,n})(\mu_j \ge \alpha)\}. \qquad (3)$$

$$(\forall B' \subset B_\alpha)[\tilde{R}(B') = \{<\mu'_j/x_j> \mid x_j \in X \& (\exists j = \overline{1,n})(\mu'_j < \alpha)\}]. \qquad (4)$$

Condition (3) designates that any vertex or is included into set B_α, or is accessible from some vertex of the same set with the degree not less than α. Condition (4) designates that some subset $B' \subset B_\alpha$ does not have condition (3).

The following property follows from definition of fuzzy base:

Property 1. In fuzzy base B_α there are no two vertices which enter into the same strong connectivity fuzzy subgraph with the degree above or equal α.

Let $\{\mu_1, \mu_2, ..., \mu_L\}$ be a set of all values of the membership function which are attributed to the edges of graph $\tilde{G}$. Then the following properties are true:

Property 2. In any fuzzy circuit-free graph there exist exactly L fuzzy bases with degrees $\{\mu_1, \mu_2, ..., \mu_L\}$.

Property 3. In any fuzzy circuit-free graph there is just one fuzzy base with degree α.

Property 4. If in a fuzzy circuit-free graph inequality $\alpha_1 < \alpha_2$ is executed, then inclusion $B_{\alpha_1} \supset B_{\alpha_2}$ is carried out.

Let us note the interrelation between fuzzy bases and strong connectivity fuzzy subgraphs. The following properties are true.

Property 5. If subset B_α is a fuzzy base with degree α, then there is such a subset $X' \subseteq X$, that $B_\alpha \subset X'$, and fuzzy subgraph $\tilde{G}' = (X', \tilde{U}')$ has the degree of strong connectivity not less than α.

Property 6. If subset B_α is a fuzzy base with degree α, then there is not such subset $X' \subseteq X$, that $X' \subseteq B_\alpha$ and fuzzy subgraph $\tilde{G}' = (X', \tilde{U}')$ has the degree of strong connectivity α.

The following consequence follows from property 6:

Consequence 1. If in fuzzy graph $\tilde{G}$ there was a fuzzy base with degree α, consisting of only one vertex, it is necessary that fuzzy graph $\tilde{G}$ has a strong connectivity with degree α.

Property 7. Let $\gamma(x_i,x_j)$ be an accessible degree of vertex x_j from vertex x_i. Then the following statement is true:

$$(\forall x_i, x_j \in B_\alpha)[\gamma(x_i, x_j) < \alpha].$$

On the contrary, the accessible degree of any vertex $x_j \in B_\alpha$ from any other vertex $x_i \in B_\alpha$ is less than meaning α.

Let a set $\tau_k = \{X_{k1}, X_{k2}, ..., X_{kl}\}$ be given, where X_{ki} is a fuzzy k-vertex base with the degree of α_{ki}. We define α_k as $\alpha_k = \max\{\alpha_{k1}, \alpha_{k2}, \ldots, \alpha_{kl}\}$. In case $\tau_k = \varnothing$ we define $\alpha_k = \alpha_{k-1}$. Volume α_k means that fuzzy graph $\tilde{G}$ includes a k-vertex subgraph with the accessible degree of domination α_k and doesn't include k-vertex subgraph with an accessible degree more than α_k.

Definition 2. A fuzzy set

$$\tilde{B} = \{<\alpha_1/1>, <\alpha_2/2>, ..., <\alpha_n/n>\}$$

is called a fuzzy set of bases of fuzzy graph $\tilde{G}$.

The fuzzy set of bases is a fuzzy invariant of a fuzzy graph. The fuzzy set of bases determines the highest degree of the reachability of the vertices for any given number of service centers.

Example 6. For the fuzzy graph presented in Fig.1, the fuzzy set of bases is

$$\tilde{B} = \{<0{,}5/1>, <0{,}6/2>, <0{,}7/3>, <0{,}8/4>, <1/5>\}.$$

The fuzzy set of bases for this graph means, in particular, that if we have 2 service centers, then at their optimal placement all the other vertices are achievable with a degree of not less than 0.6.

Property 8 The following proposition is true

$$0 \le \alpha_1 \le \alpha_2 \le ... \le \alpha_n = 1.$$

Thus, it is necessary to determine a fuzzy set of bases for finding the greatest degree of service of the given fuzzy graph vertices.

3. The method for finding bases fuzzy set

We will consider the method of finding a family of all fuzzy bases with the highest degree. The given method is similar to Maghout's method for the definition of all minimal fuzzy dominating vertex sets [10], to Maghout's method for the definition of fuzzy antibases for fuzzy graphs [12], and to Maghout's method for the definition of fuzzy vitality sets for fuzzy graphs [13].

Let us aassume that set B_α is a fuzzy base of the fuzzy graph $\tilde{G}$ with degree α. Then for an arbitrary vertex $x_i \in X$, one of the following conditions must be true.

a) $x_i \in B_\alpha$;

b) if $x_i \notin B_\alpha$, then there is a vertex x_j so that it belongs to set B_α with the degree $\gamma(x_j,x_i) \ge \alpha$.

In other words, the following statement is true:

$$(\forall x_i \in X)[x_i \in B_\alpha \vee \\ \vee (x_i \notin B_\alpha \rightarrow (\exists x_j \in B_\alpha | \gamma(x_j,x_i) \ge \alpha))]. \quad (5)$$

To each vertex $x_i \in X$ we assign Boolean variable p_i that takes value 1, if $x_i \in B_\alpha$ and 0 otherwise. We assign the fuzzy variable $\xi_{ji} = \alpha$ for the proposition $\gamma(x_j,x_i) \ge \alpha$. Passing from the quantifier form of proposition (5) to the form in terms of logical operations, we obtain a true logical proposition:

$$\Phi_B = \underset{i}{\&}(p_i \vee (\overline{p}_i \rightarrow (\underset{j}{\vee}(p_j \& \gamma_{ji})))).$$

Taking into account the interrelation between the implication operation and disjunction operation ($\alpha \rightarrow \beta = \overline{\alpha} \vee \beta$), we receive

$$\Phi_B = \underset{i}{\&}(p_i \vee p_i \vee \underset{j}{\vee}(p_j \& \gamma_{ji})).$$

Supposing $\xi_{ii} = 1$ and considering that the equality $p_i \vee \underset{j}{\vee} p_i \& \xi_{ij} = \underset{j}{\vee} p_j \xi_{ij}$ is true for any x_i, we finally obtain

$$\Phi_B = \underset{i}{\&}(\underset{j}{\vee}(p_j \& \gamma_{ji})). \quad (6)$$

We open the parentheses in the expression (6) and reduce the similar terms by following rules

$$a \vee a \& b = a; \\ a \& b \vee a \& \overline{b} = a; \\ \xi_1 \& a \vee \xi_2 \& a \& b = \xi_1 \& a \text{ if } \xi_1 \ge \xi_2. \quad (7)$$

Here, $a,b \in \{0,1\}$ and $\xi_1,\xi_2 \in [0,1]$.

Then the expression (6) may be presented as

$$\Phi_B = \underset{i=\overline{1,l}}{\vee}(p_{1_i} \& p_{2_i} \& \ldots \& p_{k_i} \& b_i). \quad (8)$$

We may prove next property:

Property 9. Each disjunctive member in the expression (8) gives a fuzzy base with the highest degree b_i.

The following method of foundation of a fuzzy set of bases may be proposed on the base of Property 9:

- We write proposition (6) for given fuzzy graph $\tilde{G}$;
- We simplify proposition (6) by proposition (7) and present it as proposition (8);
- We define all the fuzzy bases, which correspond to the disjunctive members of proposition (8);
- We define the fuzzy set of bases.

4. Example of service centers finding

Let us consider a railway network limited by the stations Novosibirsk, Kemerovo, Barnaul and Novokuzneck. The network is presented in Fig. 2.

The fuzzy graph of this railway network, obtained from the GIS "Object Land" [13], is represented in Fig. 3. Here the fuzzy graph vertices correspond to the network railway stations, and the fuzzy graph edges correspond to the rail way between the stations. The membership functions of the edges are calculated according to the characteristics of the railway. For example, "if the period of the maintenance of the railway is less than 15 years and the length is less than 20 kilometers, the membership function is equal to 0.9". It is necessary to find the allocation of service centers. For the sake of simplicity, we will present by one vertex, that all subgraphs with a strong connection degree equals 1.

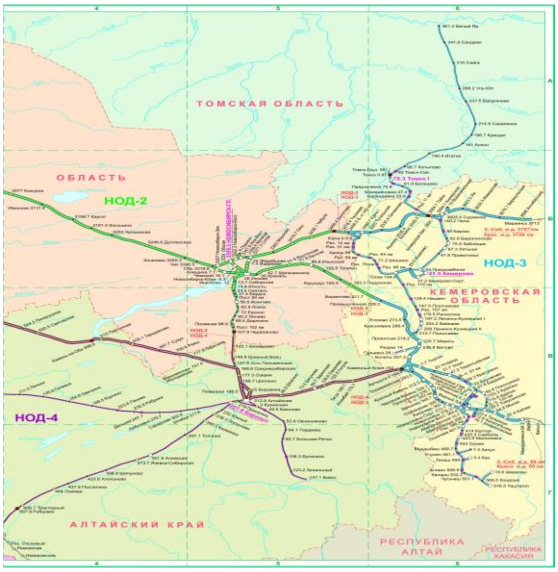

Fig. 2: Railway network.

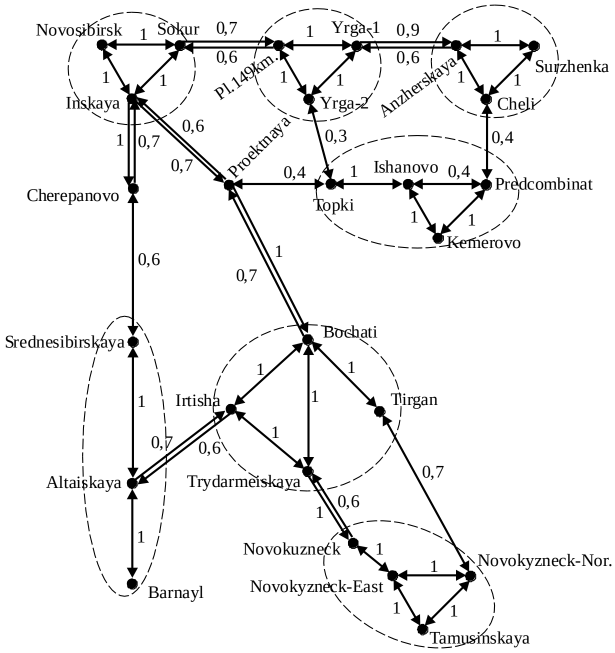

Fig. 3: Fuzzy graph of the railway network.

As a result we will receive the aggregative fuzzy graph with *n*=9, which is represented in Fig. 4:

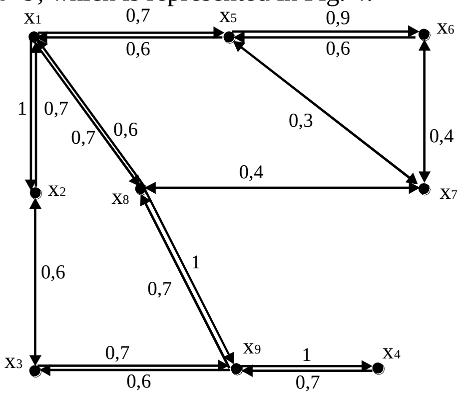

Fig. 4: Aggregative fuzzy graph of the railway network.

The vertex matrix for this graph has the following form

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{array} \\
R = \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{array} &
\left| \begin{array}{ccccccccc}
0 & 1 & 0 & 0 & 0{,}7 & 0 & 0 & 0{,}7 & 0 \\
0{,}7 & 0 & 0{,}6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}6 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}7 \\
0{,}6 & 0 & 0 & 0 & 0 & 0{,}9 & 0{,}3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0{,}6 & 0 & 0{,}4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0{,}3 & 0{,}4 & 0 & 0{,}4 & 0 \\
0{,}6 & 0 & 0 & 0 & 0 & 0 & 0{,}4 & 0 & 0{,}7 \\
0 & 0 & 0{,}6 & 1 & 0 & 0 & 0 & 0{,}7 & 0
\end{array} \right|
\end{array}
$$

We raise the contiguity matrix to 2, 3, …, 9 powers. Uniting them, we find an accessible matrix

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{array} \\
N = \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{array} &
\left| \begin{array}{ccccccccc}
1 & 1 & 0{,}7 & 0{,}7 & 0{,}7 & 0{,}7 & 0{,}4 & 0{,}7 & 0{,}7 \\
0{,}7 & 1 & 0{,}6 & 0{,}7 & 0{,}7 & 0{,}7 & 0{,}4 & 0{,}7 & 0{,}7 \\
0{,}6 & 0{,}6 & 1 & 0{,}7 & 0{,}6 & 0{,}6 & 0{,}4 & 0{,}7 & 0{,}7 \\
0{,}6 & 0{,}6 & 0{,}6 & 1 & 0{,}6 & 0{,}6 & 0{,}4 & 0{,}7 & 0{,}7 \\
0{,}6 & 0{,}6 & 0{,}6 & 0{,}6 & 1 & 0{,}9 & 0{,}4 & 0{,}6 & 0{,}6 \\
0{,}6 & 0{,}6 & 0{,}6 & 0{,}6 & 0{,}6 & 1 & 0{,}4 & 0{,}6 & 0{,}6 \\
0{,}4 & 0{,}4 & 0{,}4 & 0{,}4 & 0{,}4 & 0{,}4 & 1 & 0{,}4 & 0{,}4 \\
0{,}6 & 0{,}6 & 0{,}6 & 1 & 0{,}6 & 0{,}6 & 0{,}4 & 1 & 1 \\
0{,}6 & 0{,}6 & 0{,}6 & 1 & 0{,}6 & 0{,}6 & 0{,}4 & 0{,}7 & 1
\end{array} \right|
\end{array}
$$

The corresponding expression (6) for this graph has the following form

$$
\begin{aligned}
\Phi_B = {} & (1p_1 \vee 0{,}7p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 0{,}6p_8 \vee 0{,}6p_9)\,\& \\
& \&\,(1p_1 \vee 1p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 0{,}6p_8 \vee 0{,}6p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}6p_2 \vee 1p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 0{,}6p_8 \vee 0{,}6p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}7p_3 \vee 1p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 1p_8 \vee 1p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 1p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 0{,}6p_8 \vee 0{,}6p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}9p_5 \vee 1p_6 \vee 0{,}4p_7 \vee \\
& \vee 0{,}6p_8 \vee 0{,}6p_9)\,\& \\
& \&\,(0{,}4p_1 \vee 0{,}4p_2 \vee 0{,}4p_3 \vee 0{,}4p_4 \vee 0{,}4p_5 \vee 0{,}4p_6 \vee 1p_7 \vee \\
& \vee 0{,}4p_8 \vee 0{,}4p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}7p_3 \vee 0{,}7p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 1p_8 \vee 0{,}7p_9)\,\& \\
& \&\,(0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}7p_3 \vee 0{,}7p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee \\
& \vee 1p_8 \vee 1p_9).
\end{aligned}
$$

In the received expression the first parenthesis absorbs the second parenthesis, the eighth parenthesis absorbs the ninth parenthesis. Multiplying parenthesis 3 and 4, parenthesis 5 and 6, parenthesis 7 and 8, and using rules (7) we obtain

$$\Phi_B = (1p_1 \vee 0{,}7p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee$$
$$\vee 0{,}6p_8 \vee 0{,}6p_9)\ \&$$
$$\&\ (0{,}7p_1 \vee 0{,}6p_2 \vee 0{,}7p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee$$
$$\vee 0{,}6p_8 \vee 0{,}6p_9 \vee 1p_3p_4 \vee 1p_3p_8 \vee 1p_3p_9)\ \&$$
$$\&\ (0{,}7p_1 \vee 0{,}7p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}9p_5 \vee 0{,}6p_6 \vee 0{,}4p_7 \vee$$
$$\vee 0{,}6p_8 \vee 0{,}6p_9 \vee 1p_5p_6)\ \&$$
$$\&\ (0{,}4p_1 \vee 0{,}4p_2 \vee 0{,}4p_3 \vee 0{,}4p_4 \vee 0{,}4p_5 \vee 0{,}4p_6 \vee 0{,}4p_7 \vee$$
$$\vee 0{,}4p_8 \vee 0{,}4p_9 \vee 0{,}7p_1p_7 \vee 0{,}7p_2p_7 \vee 0{,}7p_3p_7 \vee 0{,}7p_4p_7 \vee$$
$$\vee 0{,}6p_5p_7 \vee 0{,}6p_6p_7 \vee 1p_7p_8 \vee 0{,}7_7p_9).$$

Multiplying parenthesis 1 and 2, parenthesis 2 and 4 and using rules (7) we obtain

$$\Phi_B = (0{,}7p_1 \vee 0{,}6p_2 \vee 0{,}6p_3 \vee 0{,}6p_4 \vee 0{,}6p_5 \vee 0{,}6p_6 \vee$$
$$\vee 0{,}4p_7 \vee 0{,}4p_8 \vee 0{,}6p_9 \vee 0{,}7p_2p_3 \vee 1p_1p_3p_4 \vee$$
$$\vee 1p_1p_3p_8 \vee 1p_1p_3p_9)\ \&$$
$$\&\ (0{,}4p_1 \vee 0{,}4p_2 \vee 0{,}4p_3 \vee 0{,}4p_4 \vee 0{,}4p_5 \vee 0{,}4p_6 \vee$$
$$\vee 0{,}4p_7 \vee 0{,}4p_8 \vee 0{,}4p_9 \vee 0{,}7p_1p_7 \vee 0{,}7p_2p_7 \vee 0{,}6p_3p_7 \vee$$
$$\vee 0{,}6p_4p_7 \vee 0{,}6p_5p_7 \vee 0{,}6p_7p_8 \vee 0{,}6p_7p_9 \vee 0{,}7p_4p_5p_7 \vee$$
$$\vee 0{,}9p_5p_7p_8 \vee 0{,}7p_5p_7p_9 \vee 0{,}7p_3p_5p_6p_7 \vee 1p_5p_6p_7p_8 \vee$$
$$\vee 0{,}7p_5p_6p_7p_9).$$

Multiplying the parentheses, we finally obtain

$$\Phi_B = \underline{0{,}4p_1} \vee 0{,}4p_2 \vee 0{,}4p_3 \vee 0{,}4p_4 \vee 0{,}4p_5 \vee 0{,}4p_6 \vee$$
$$\vee 0{,}4p_7 \vee 0{,}4p_8 \vee 0{,}4p_9 \vee \underline{0{,}7p_1p_7} \vee 0{,}6p_2p_7 \vee$$
$$\vee 0{,}6p_3p_7 \vee 0{,}6p_4p_7 \vee 0{,}6p_5p_7 \vee 0{,}6p_7p_9 \vee 0{,}7p_2p_3p_7 \vee$$
$$\vee \underline{0{,}9p_1p_3p_5p_7p_9} \vee \underline{1p_1p_3p_5p_6p_7p_8}.$$

It follows from the last equality that graph $\tilde{G}$ has 18 fuzzy bases, and the fuzzy set of bases is defined as:

$$\tilde{B} = \{<0{,}4/1>, <0{,}7/2>, <0{,}9/5>, <1/6>\}.$$

The fuzzy set of bases defines the following optimum allocation of the service centres: If we have 6 or more service centres then we must place these centres into vertices 1, 3, 5, 6, 7 and 8 (Inskaya, Barnaul, Yrga-2, Surzhenka, Kemerovo, Proektnaya). The degree of service equals 1 in this case. If we have 5 service centres then we must place these centres into vertices 1, 3, 5, 7 and 9 (Inskaya, Barnaul, Yrga-2, Kemerovo, Tirgan). In this case the degree of service equals 0.9. If we have 2 service centres then we must place both centres into vertices 1 and 7 (Inskaya, Kemerovo). In this case the degree of service equals 0.7. If we have only one service centre then we can place it in any vertex (for example, Inskaya). In the last case the degree of service equals 0.4.

5. Conclusion

The task of defining of optimal allocation of centres as the task of definition fuzzy bases of fuzzy graphs was considered. The definition method of fuzzy bases is the generalization of Maghout's method for nonfuzzy graphs. This method is effective for the graphs which have no homogeneous structure and no large dimensionality. It is necessary to point out that the considered task may be solved by the multi-objective performance. In our future work we are going to investigate the task of the service centers accommodation on the fuzzy temporal graphs, i.e. such graphs, whose edges membership functions change in discrete time, will also be studied.

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