

E-W-CM:	$\frac{\varphi \triangleright \gamma ; \varphi' \triangleright \gamma}{(\varphi \wedge \varphi') \triangleright \gamma}$	$\mu_\gamma \preceq \mu_{Expl(\varphi)} ; \mu_\gamma \preceq \mu_{Expl(\varphi')}$
E-W-C-Cut:	$\frac{(\varphi \wedge \varphi') \triangleright \gamma ; \forall \delta [\text{if } \varphi \triangleright \delta \text{ then } \varphi' \triangleright \delta]}{\varphi \triangleright \gamma}$	$\frac{\mu_\gamma \preceq \mu_{Expl(\varphi \wedge \varphi')}, \mu_\gamma \preceq \mu_{Expl(\varphi)}}{\mu_\gamma \preceq \mu_{Expl(\varphi \wedge \varphi')}} ; \forall \delta [\text{if } \mu_\delta \preceq \mu_{Expl(\varphi)} \text{ then } \mu_\delta \preceq \mu_{Expl(\varphi')}]$

Table 7: Weak forms of some rationality postulates, expressed in syntactic and semantic forms.

syntactic and semantic expressions are given in Table 7. A weak version of E-R-Cut can be defined in a similar way.

Proposition 8 *The explanatory relation \triangleright^{ene} derived from fuzzy erosions with any structuring element B satisfies E-W-CM and E-W-C-Cut.*

6. Conclusion

New explanatory relations have been proposed for knowledge representation based on logics with fuzzy sets of models, thus accounting with the approximate nature of abductive reasoning. The algebraic properties of the involved mathematical morphology operators lead to good properties of the proposed relations in terms of rationality properties. Future work aims at further developing examples, at investigating the potential role of α for balancing specialization and generalization of the solution, and at extending the formalism to other types of fuzzy logics.

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