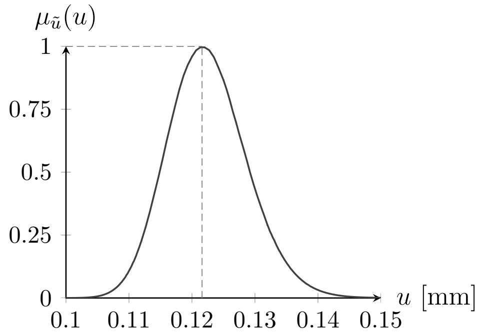

Figure 6: Fuzzy displacement $\tilde{u}$ of the tip of the rod.

The standard deviations of $\tilde{c}_1$ and $\tilde{c}_2$ are assumed to be 5 % of their modal values. Hence, the parametric representation of the fuzzy stiffness parameters is

$$\tilde{c}_i = \mathrm{gfn}(\bar{c}_i, 0.05\,\bar{c}_i, 0.05\,\bar{c}_i), \quad i = 1, 2.$$

In order to compute the α-cuts of the fuzzy displacement $\tilde{u}$, we consider again Eq. (3), where we can see that u is (strictly) monotonic decreasing in both c_1 and c_2 for positive values. Hence, the α-cuts $u(\alpha) = [u^{\mathrm{L}}(\alpha), u^{\mathrm{R}}(\alpha)]$ of $\tilde{u}$ are

$$u^{\mathrm{L}}(\alpha) = u\big(c_1^{\mathrm{R}}(\alpha), c_2^{\mathrm{R}}(\alpha)\big) = \frac{1007}{414\left(20 + \sqrt{-2\ln(\alpha)}\right)},$$
$$u^{\mathrm{R}}(\alpha) = u\big(c_1^{\mathrm{L}}(\alpha), c_2^{\mathrm{L}}(\alpha)\big) = \frac{1007}{414\left(20 - \sqrt{-2\ln(\alpha)}\right)},$$

and its membership function yields

$$\mu_{\tilde{u}}(u) = \exp\left[-\frac{1}{a}\left(b - \frac{c}{u} + \frac{d}{u^2}\right)\right], \quad u > 0,$$

where

$$a = 342792, \qquad b = 68558400,$$
$$c = 16675920, \quad d = 1014049.$$

The plot of $\mu_{\tilde{u}}(u)$ in the range $0.1 \leq u \leq 0.15$ is illustrated in Figure 6.

6. Conclusions

The proposed analytical approach turns out to be a very practical tool for the inclusion of parameter uncertainties into mathematical models. It is valid for continuous, monotonic functions of independent fuzzy numbers, but can also be applied to fuzzy intervals as defined, e. g., in [3].

An analytical solution has the advantage that the degrees of membership of the fuzzy output can be computed for any value within the support, whereas a numerical solution only provides a finite number of values. Furthermore, our approach also allows a symbolic processing of uncertainties.

In further research activities, this approach shall be extended to general, non-monotonic functions of independent fuzzy numbers, where the influence of interdependency shall be investigated as well.

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