When we consider A we can find a permutation such that $A = A_{i_0}$ hence $\Delta(A) = \nu^c(A_{i_0}) = \nu^c(A)$.

$\eta(\Delta(\overline{A})) = \nabla(A) = \eta(\nu^c(\overline{A})) = \nu(A)$

$$\oint_{\nu}^{\lightning}(f) = \oint_{\nabla}^{\lightning}(f) \geq \bigwedge_{\delta \in \mathcal{S}(\nu^c)} \oint_{\nabla}^{\lightning}(f). \qquad (10)$$

If $\delta \in \mathcal{S}(\nu^c)$ then $\eta(\nu^c(\overline{A})) \leq \eta(\Delta(\overline{A}))$ i.e $\nu(A) \leq \nabla(A)$. So $\oint_{\nu}^{\lightning}(f) \leq \oint_{\nabla}^{\lightning}(f)$ for all $\delta \in \mathcal{S}(\nu^c)$. So $\oint_{\nu}^{\lightning}(f) \leq \bigwedge_{\delta \in \mathcal{S}(\nu^c)} \oint_{\nabla}^{\lightning}(f)$.

The qualitative desintegrals can also be expressed as lower or upper bounds using the relations between the qualitative integrals and desintegrals. This may be used for obtaining the results for desintegrals from the ones regarding integrals.

$\oint_{\nu}^{\lightning}(f) = \mathcal{S}_{\nu(\bar{\cdot})}(\eta(f)) = \bigwedge_{\pi \in S(\nu(\bar{\cdot}))} \mathcal{S}_{\Pi}(\eta(f))$.
$\oint_{\nu}^{\downarrow}(f) = \oint_{\eta(\nu)}^{\uparrow}(\eta(f)) = \bigvee_{\pi \in S(\eta(\nu))} \oint_{\Pi}^{\uparrow}(\eta(f))$.
$\oint_{\nu}^{\Downarrow}(f) = \oint_{\nu(\bar{\cdot})}^{\Uparrow}(\eta(f)) = \bigwedge_{\pi \in S(\nu(\bar{\cdot}))} \oint_{\Pi}^{\Uparrow}(\eta(f))$.

For instance, Proposition 13 can be obtained by noticing that $\oint_{\nu}^{\lightning}(f) = \mathcal{S}_{\nu(\bar{\cdot})}(\eta(f)) = \bigwedge_{\pi \in S(\nu(\bar{\cdot}))} \mathcal{S}_{\Pi}(\eta(f)) = \bigwedge_{\eta(\delta) \in S(\nu(\bar{\cdot}))} \mathcal{S}_{\nabla}(f)$
since $\nabla_{\delta}(A) = \Pi_{\eta(\delta)}(\overline{A})$. It can be checked that $\pi \in S(\eta(\nu)) \Leftrightarrow \eta(\pi) \in \mathcal{S}(\nu)$, and thus
$\eta(\delta) \in S(\nu(\bar{\cdot})) \Leftrightarrow \delta \in \mathcal{S}(\eta(\nu(\bar{\cdot}))) \Leftrightarrow \delta \in \mathcal{S}(\nu^c)$, which completes the checking of Proposition 13.

7. Conclusion

Retrospectively, one may wonder why one has only considered Sugeno integrals for a long time, since the other qualitative integrals are as simple. The results obtained in this paper makes Sugeno integrals, and other integrals or desintegrals easier to compute as a simple combination of integrals or desintegrals with respect to possibility, necessity, guaranteed possibility measures (i.e. basically weighted max and min). Besides, the representation of capacities as a finite conjunction of possibility measures, or as a finite disjunction of necessity measures has strong links with k-maxitivity and k-minitivity axioms, and the representation of imprecise possibilities by means of possibilistic focal elements of limited size. This will motivate further investigation.

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