Interpretability analysis of fuzzy association rules supported by fingrams

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Abstract

This work extends fuzzy inference-grams (fingrams) to fuzzy association rules (FAR), yielding FAR-Fingrams. Their analysis pays attention to interpretability issues. An important open problem in association rule mining is the huge number of frequent itemsets and interesting rules to uncover and communicate to the user. FAR-Fingrams address such problem through visual analysis. They ease the selection of rules according to the user's preferences and quality criteria. A new metric to construct fingrams is proposed, reflecting the particularities of FAR. Finally, some of the potentials of FAR-Fingrams are overviewed over a real-world problem that deals with user-preferences.

Keywords: Interpretability, fuzzy association rules, rule co-firing, social network analysis.

1. Introduction

Interpretability is an appreciated and distinguished capability of fuzzy systems [1]. However, it is important to highlight the fact that fuzzy systems are not interpretable *per se*. Of course, the high semantic expressivity (close to natural language) of linguistic variables [2] and rules [3, 4] favors interpretability, but only a careful design guarantees the fulfillment of interpretability requirements [5].

Fuzzy association rules [6] permit the uncovering of dependencies among items in datasets. They have been successfully applied to a wide variety of problems [7] such as, effective fuzzy associative classification [8].Unfortunately, systems made up of automatically extracted fuzzy association rules are rarely as interpretable as desired [8]. A large number of complex rules, that involve many different variables, usually compounds such systems, making them quite hard to interpret.

More interpretable fuzzy association rule systems can be obtained by reducing the number of rules [9]. However, this solution usually generalizes not maintaining exactly the same system behavior. Obviously, a tool supporting the evaluation and comprehension of rules could help in the identification of valuable rules.

Fuzzy inference-grams (fingrams) support the interpretability-driven design of fuzzy systems [10]. Fingrams simplify system analysis from a comprehensibility point of view, providing graphical representations of the inference layer of fuzzy systems [11]. Thus, the interactions among hundreds of fuzzy rules are displayed in the form of close-to-tree graphs easy to interpret. Such graphs are seen and analyzed as social networks which comprise nodes representing rules and edges showing rule co-firing. Moreover, fingrams are not affected by the well-known curse of dimensionality characteristic of fuzzy systems. Therefore, experts can comfortably analyze fingrams, even if they include a large number of rules, with the aim of understanding the structure and behavior of the represented fuzzy system. This paper explains how to use effectively an extension of fingrams for the representation and analysis of fuzzy association rules.

The rest of the contribution is organized as follows. Sec. 2 presents some preliminaries of fuzzy association rules (FAR) and fuzzy inference-grams (Fingrams). Sec. 3 details the particularities of FAR-Fingrams. Sec. 4 presents a case study that sketches their main benefits. Finally, Sec. 5 points out some conclusions and future works.

2. Preliminaries

2.1. Fuzzy Association Rules (FAR)

Association rules identify and represent dependencies among items in a dataset [12]. They are representations of the type $X \rightharpoonup Y$, being X and Y itemsets such as $X \cap Y = \oslash$. Therefore, if the items in X exist in a pattern then it is likely that the items in Y are also present in the pattern. In addition, X and Y should not have items in common [13]. For instance, in the case of market basket analysis the association $\{digital\ camera, battery\} \rightharpoonup \{memory\ card\}$ suggests a purchase containing a digital camera and a battery usually includes a memory card.

A high number of previous studies on mining association rules focused on datasets with discrete or binary values; however, in real-world applications, data usually consists of quantitative values. Defining Data Mining algorithms able to deal with various types of data is a great challenge. Fuzzy sets have been widely used in Data Mining with that aim, mainly because of its similarity to human reasoning [6]. The use of fuzzy sets avoids unnatural boundaries in the partitioning of the attribute domains and improves the interpretation of rules in natural language. There are many methods for mining fuzzy association rules from datasets with quantitative values [7, 14, 15]. Considering a simple dataset with two attributes (A_1 and A_2) and three linguistic terms (LOW, MIDDLE, HIGH). An illustrative example of fuzzy association rule is A_1 *is* HIGH $\rightharpoonup$ A_2 *is* MIDDLE.

Regarding assessment of fuzzy association rules, *Support* and *Confidence* are widely admitted as the most popular measures:

$$Support(X \rightharpoonup Y) = \frac{\sum_{x_p \in D} \mu_{XY}(x_p)}{\mid N \mid} \quad (1)$$

$$Confidence(X \rightharpoonup Y) = \frac{\sum_{x_p \in D} \mu_{XY}(x_p)}{\sum_{x_p \in D} \mu_{X}(x_p)} \quad (2)$$

where $\mu_X(x_p)$ is the matching degree of the pattern x_p with the antecedent of the rule; $\mu_{XY}(x_p)$ is the matching degree of the pattern x_p with the antecedent and consequent of the rule; and $\mid N \mid$ is the cardinality of the dataset D.

The classic association rule mining techniques look for rules with *Support* and *Confidence* greater than the user-defined thresholds *minimum support* and *minimum confidence*. However, this approach yields many more rules than expected [16]. Therefore, other quality measures for the selection and ranking of patterns, according to their potential interest to the user, have been proposed [17]. For instance, *Lift* [18]:

$$Lift(X \rightharpoonup Y) = \frac{Confidence(X \rightharpoonup Y)}{\sum_{x_p \in D} \mu_{Y}(x_p) / \mid N \mid} \quad (3)$$

which represents the ratio between the actual rule confidence and the expected one. This measure takes values in [0,∞), detecting negative dependence ($Lift < 1$), positive dependence ($Lift > 1$), or independence ($Lift = 1$) among items.

2.2. Fuzzy inference-grams (Fingrams)

The fuzzy inference-grams, fingrams in short, can be seen as social networks that provide a graphical view of fuzzy systems at inference level. Nodes represent fuzzy rules, while the interactions among rules are represented by weighted edges whose value is computed using a specific metric, typically a rule co-firing metric. Fingrams were firstly introduced in [19]. Recently, we proposed a methodology for visual representation and exploratory analysis of fuzzy rule-based systems based on fingrams [11]. It comprises the following steps:

1. **Network generation**: Given a dataset, a rule base made up of r rules, a fuzzy reasoning mechanism and a rule co-firing metric m, then the complete set of relations among rules is inferred, yielding an initial social network defined by a $r \times r$ square matrix M. Each element m_{ij} characterizes the degree of interaction between rules r_i and r_j. $m_{ij} = 0$ means there is not any data sample firing at the same time both rules.

2. **Network scaling**: Since the previous network is usually very dense and difficult to analyze, a scaling process is required. As result, we obtain a simplified social network highlighting the most relevant relations among rules while preserving the backbone of the initial network. We use Pathfinder algorithm [20] to scale the original social network. Fast Pathfinder [21], a faster variant of the original, is used in this approach.

3. **Network drawing**: A layout algorithm, guided by aesthetical criteria, automatically places the nodes and edges of the scaled network. Kamada-Kawai algorithm, through Graphviz[1], is used in this contribution due to the demonstrated effectiveness in combination with Pathfinder [22].

4. **Network visualization**: The user can analyze the final graph (what we call fingram), studying the general structure of the resultant network or zooming out a particular subset of nodes. The analysis of fingrams is supported by social network analysis techniques (centrality-based analysis techniques, community mining techniques, and so on). This offers many valuable possibilities, such as, understanding the structure and behavior of the related fuzzy rule base, uncovering the most significant rules according to specific criteria, etc.

3. FAR-Fingrams

Fingrams for fuzzy association rules, in short FAR-Fingrams, are aimed at facilitating the visual analysis and comprehension of fuzzy association rules. We have adapted fingrams, originally designed for classification and regression fuzzy rule based systems [11], to deal with the particularities of fuzzy association rules.

In the original fingrams, the size of nodes is proportional to the number of examples covered by the related rules. Dually, in FAR-Fingrams the node size is determined by the *Support* of the corresponding rule, because it plays a central role in the assessment of fuzzy association rules [23]. Even more, as it happens in the original fingrams, the number of node borders shows the number of rule antecedents.

[1] *http://www.graphviz.org/*

No matter the kind of fuzzy rules that are considered, fingram nodes are labeled with relevant textual information. The first line always indicates the rule identifier while the rest of lines are customized for each rule type. In the particular case of FAR-Fingrams they correspond to quality measures. The second line gives the *Support*, the third line shows the *Confidence*, and *Lift* is given in the last line.

Regarding the color of nodes, it is linked to rule outputs in the original fingrams. For instance, the same color is given to all rules pointing out the same class in classification problems. Unfortunately, fuzzy association rules can have a huge variety of potential outputs what makes unfeasible to differentiate nodes by this criterion. Thus, we propose to use a grey scale to indicate the *Lift* level of rules which yields an idea about the goodness of the selected rule. The higher the *Lift*, the darker the node background.

The fuzzy association rule mining heavily exploits the structure of patterns which is reflected naturally in form of generalization/specialiation relations. With the aim of taking this fact into account we have defined a new asymmetric co-firing metric for FAR-Fingrams:

$$m_{ij} = 1 - \frac{\sum_{x \in X} (|FD_i(x) - FD_j(x)|)}{\sum_{x \in X} FD_i(x)} \qquad (4)$$

with $x \in X$ being all the examples firing rule R_i; $FD_i(x)$ is the firing degree up to which a single example x fires the rule R_i.

This new co-firing metric characterizes generalization/specialization relations between pairs of rules. Moreover, it yields a directed graph, i.e., each edge has associated two possible arrows (one per direction). In case both arrows have the same weight then they are substituted by one undirected link. Notice that edge thickness is proportional to the related weight m_{ij}. Rule R_i is highly related with R_j, i.e. $R_i \xrightarrow{\approx 1} R_j$, when R_j is fired at similar degrees by the same set of examples that fires R_i.

In addition, the visual analysis of FAR-Fingrams allows uncovering the behavior of individual rules. Size and darkness of nodes inform about *Support* and *Lift*. *Confidence* is given in the textual information related to each single node. Thus, FAR-Fingrams show simultaneously information related to several quality measures. Obviously, this is much more effective than considering only one-ranking evaluation guided by a single measure as usual.

Interestingly, the analysis permits also detecting some common behaviors in groups of fuzzy association rules. The way how rules are connected to each other gives an idea of the complexity and interrelation among them. Spare rules are usually easier to understand, whereas dense connected rules are more complex to comprehend. However, there exist examples that produce complete subgraphs with very high relations what reflects a common behavior:

- Highly related rules are candidates to be merged into a more general one because they normally share most of the antecedents and consequents. They are easily recognized as very dense structures.
- Isolated sets of rules covering disjoint sets of examples appears like islands. Those connected rules inside the island do not share any example with other external rules.

4. Case study

The utility of FAR-Fingrams can be illustrated in a real-world problem dealing with qualitative assessment of industrial objects automatically designed through cognitive engineering. Namely, we focus on finding out the most interesting fuzzy association rules related to explain how different users evaluated the degree of femininity of a set of chairs. We considered data coming from a project[2] where people had to evaluate the femininity degree of 23 models of chairs. They were sequentially displayed, in a poll, allowing the users to introduce their appreciations in turn (see two examples of chairs in Fig. 1). The poll received 644 evaluations from 28 users (11 males and 17 females).

(a) Model with femininity "high".

(b) Model with femininity "low".

Figure 1: Examples of chairs used in the poll.

It is possible to relate the femininity degree associated to each chair with its physical properties. With that aim, once analyzed all collected data, we decided to induce fuzzy association rules from the aggregated answers provided by different groups of users. In this paper, for the sake of clarity we will discuss only the analysis for the group of women who participated in the poll. Notice that, we use this real case study just as an illustrative example, giving an overview of the potentials of FAR-Fingrams. The main goal is to detect and analyze subsets of fuzzy association rules from an expert analysis point of view.

The learning algorithm used [24] extracted both membership functions (MFs) and fuzzy association

[2]http://bit.ly/YBWJTx

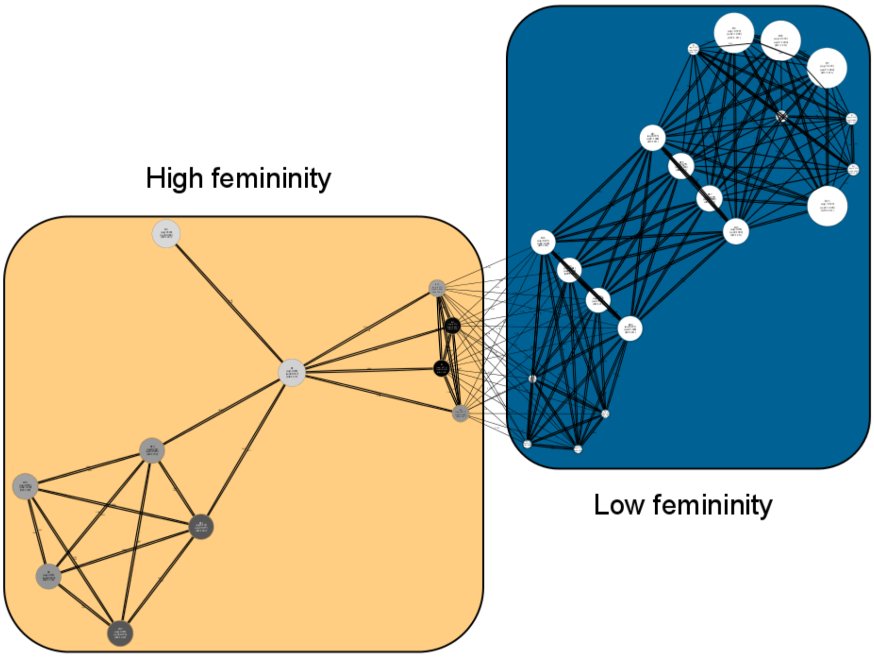


Figure 2: Complete FAR-Fingram (scaled with Pathfinder).

rules for the given dataset. It tackles with quantitative values by means of a genetic learning of the MFs based on the 2-tuples linguistic representation model and the use of a basic method for mining the fuzzy association rules. The initial linguistic partitions are comprised by 5 linguistic terms with uniformly distributed triangular MFs. The parameters of the algorithm were selected according to the recommendations of the authors, which are the default parameter settings included in the KEEL software tool [25]. Notice that, in this case we have used 0.25 and 0.9 for the minimum support and minimum confidence, respectively.

As result, we generated 31 rules that relate the variables *Femininity*, *Distance between legs*, *Distance between armrests*, *Distance from the seat to the ground*, *Type of base*, and *Type of structure*. Then, we built FAR-Fingrams to represent and analyze them. We performed several kind of filtering to uncover the most interesting rules, thus illustrating the potentials of FAR-Fingrams. Anyway, as previously mentioned, fingrams are not affected by the curse of dimensionality, and they can deal with a large amount of rules.

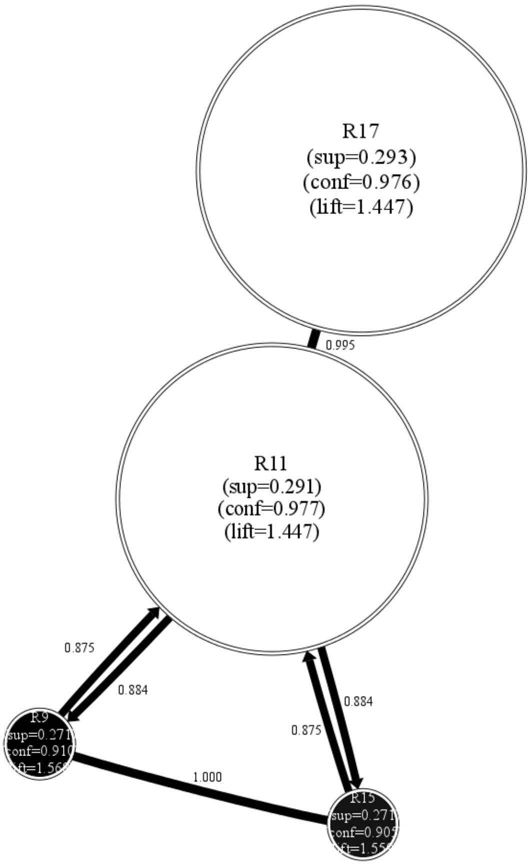
First of all, we constructed the complete FAR-Fingram[3] regarding all the 31 rules (Fig. 2). The structure of this fingram reflects a clear separation among two groups of rules, those dealing with high femininity (left hand side of the figure) and those corresponding to low femininity (right hand side). Notice that weights of edges connecting rules inside each group are much greater than the weights of those edges connecting rules belonging to the two distinct groups. This is something really close to the so-called community mining in social network analysis. A group of nodes forms a community when inner connections among group members are stronger than outer connections with members of other groups. So, we can say that the two identified groups of rules form two well defined communities according to social network analysis. Moreover, rules containing high femininity have higher *Lift* too. This fact is reflected with darker nodes.

Then, we conducted a detail analysis of each community in the quest for the most interesting rules regarding high or low femininity. In both cases, we discarded rules with more than 2 antecedents (giving priority to more general and shorter rules, from the interpretability viewpoint) and with lower *Lift*. We actually discarded those rules with *Lift* under the thresholds 1.44 and 1.21 in high and low femininity rules respectively. Figs. 3(a) and 3(b) show the resultant FAR-Fingrams. They include the rule descriptions at the bottom.

Fig. 3(a) permits appreciating that rules R9 and R15 cover the same examples with the same levels of firing (the related edge weight equals 1.0). Looking carefully at the rule description, it is easy to deduce that variable *Distance between legs* is not changing the firing degree of the handled examples. Moreover, R9 and R15 have lower *Support* (smaller nodes) and higher *Lift* (darker nodes) with respect to rules R11 and R17. In addition, we can see that all the rules are very similar and highly related. Therefore, we can look for the most interesting one considering measures of *Support*, *Confidence* and *Lift*, and highlight our selection. To do so, we have remarked, in bold, R17.

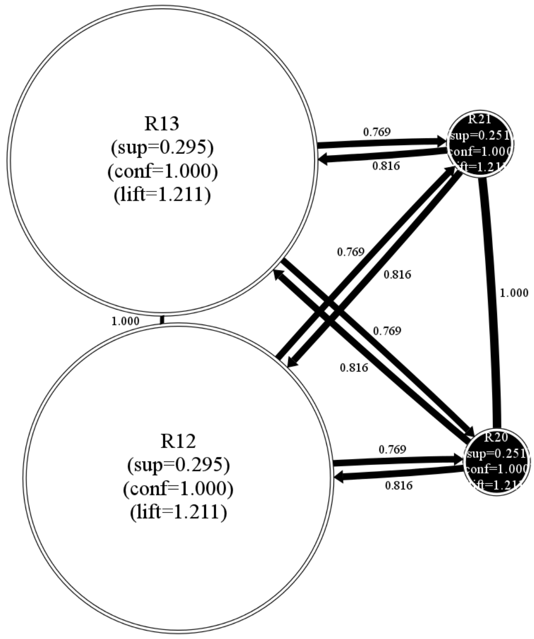
A dual analysis of Fig. 3(b) lead us to highlight

[3]All the fingrams of this contribution are built using Fingrams Generator command-line tool [26] available in *https://sourceforge.net/projects/fingrams/*



R9: {Distance between legs is Very Low AND Femininity is High} ⇀ {Distance between armrests is Very Low}
R11: {Distance between legs is Very Low AND Femininity is High} ⇀ {Distance between armrests is Low}
R15: {Distance between legs is Low AND Femininity is High} ⇀ {Distance between armrests is Very Low}
R17: {Distance between legs is Low AND Femininity is High} ⇀ {Distance between armrests is Low}

(a) Filtered fingram and rules using only rules with femininity "high", less than 3 antecedents and higher lift.



R12: {Distance between legs is Very Low AND Femininity is Low} ⇀{Base is Traditional}
R13: {Distance between legs is Very Low AND Femininity is Low} ⇀ {Structure is Geometric lines}
R20: {Distance between armrests is Low AND Femininity is Low} ⇀ {Base is Traditional}
R21: {Distance between armrests is Low AND Femininity is Low} ⇀ {Structure is Geometric lines}

(b) Filtered fingram and rules using only rules with femininity "low", less than 3 antecedents and higher lift.

Figure 3: Filtered fingrams.

R12 and R13 as the most interesting rules among those ones related to low femininity. Moreover, pairs of rules R12–R13 and R20–R21 are covering exactly the same examples. Paying attention to the symmetrical structure in Fig. 3(b) we see that R12 and R13 are equivalent. They emphasize a strong relation between low femininity and both traditional base and structure with geometric lines.

5. Conclusions

This paper has introduced FAR-Fingrams as a powerful tool to represent and analyze fuzzy association rules. We have presented a novel meaningful cofiring metric to show the relations among this kind of rules, facilitating their interpretation and related analysis. Moreover, we have proposed a particular representation of information, highlighting the most relevant features of fuzzy association rules, and including the most popular quality measures. We have illustrated the potentials of FAR-Fingrams in a case study with real data.

In the future, new metrics will be proposed, trying to depict complementary information. Moreover, new visual artifacts will be added to simplify and/or complete the final visual representation. In addition, fingrams will be extended to deal with other kind of rule-based systems.

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