Lemma 32 *Distances* $H_{\mu_m}(\nu^2, \nu^p)$ *and* $H_{\mu_{m'}}(\nu^2, \nu^p)$ *are increasing with respect to* p*, and the following holds*

- $H_{\mu_m}(\nu^2, \nu^p) \in [0, 1]$ *for all* $p \geq 2$*,*
- $H_{\mu_{m'}}(\nu^2, \nu^p) \in [0, 1]$ *for all* $p > 0$*.*

4. Conclusions

In this paper we have extended the definition of the Hellinger distance, which was initially defined for additive measures, to fuzzy measures. This extension relies on a Radon-Nikodym-type derivative for fuzzy measure.

As future work we plan to study some properties of this distance, and also study how this extension can be applied to other f-divergences, and if a general definition of f-divergence can also be given.

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