

A Multi-Expert Fuzzy TOPSIS-based Model for the Evaluation of e-Learning Paths

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Abstract

In e-learning settings, the evaluation of different alternatives regarding learning paths' proposals is nowadays crucial, due to the great attention devoted to the construction of learning objects (LO) available through Learning Management Systems (LMS). In this paper, we present a model aiming to support this evaluation process, in presence of multiple attributes and of a panel of experts involved in educational processes.

The evaluation of alternatives of e-learning paths, is carried out by each expert using the TOPSIS method under the assumption that the scores are linguistically assessed and represented by positive triangular fuzzy numbers. Given the individual rankings of alternatives, a consensus modelling mechanism is introduced where the disagreement between the rankings of single experts and the group ranking is measured with a Spearman foot rule distance. A compromise solution (consensual group ranking) is determined through a constrained optimization model where the objective function is represented by a OWA-based aggregation of individual distances and constraints are imposed on individual disagreements.

Keywords

fuzzy TOPSIS, OWA aggregation, group consensus, learning management systems, e-learning paths

1. Introduction

This paper presents a model for supporting decisions that to take in complex learning environments, where multiple proposals for e-learning paths are available to decision makers. The importance of choosing correctly the educational material for training people at distance is a central element today not only for educational institutions. Learning processes are implemented usually through the interaction of the learner with a Learning Management System (LMS), and, in some cases, through the usage of learning, or e-learning, paths.

A learning path, as referred inside a LMS, is represented by a set of Learning Objects (LOs) mixed with other tools and services available in the LMS, like

questionnaires, forums, wikis, FAQs etc., This combination of information chunks and services is devoted to obtain the educational objectives defined by an instructional designer.

While testing large scale implementation of the virtual communities system developed by our group for educational purposes [1], we noticed that learning objects created according to SCORM standard [2] are more and more important in educational settings today. The market is responding to this request, thanks to adequate technologies for the design, realization and delivery of these pre-constructed educational tools. SCORM packages themselves, if well designed, could be self-consistent learning paths.

According to this scenario, educational institutions and specifically the industry rather than academy, are very often facing the process of evaluating different possible learning paths, composed by different learning objects, based on multiple contents and representing different approaches and responses to the educational needs stated by the educational stakeholders.

The criteria for choosing which alternative better fits these needs are mostly based on simple considerations (mainly cost of the learning objects), taken by people with no complete view of different aspects of the learning paths, not taking into consideration all the aspects that should be needed for such an important step.

E-learning has many advantages, but for sure the best application field for these advantages is when large numbers of users are involved in training activities. Under these conditions, a wrong choice about the learning paths offered could therefore have serious consequences.

In order to support the decision making process aiming at selecting the most suitable e-learning path(s), we introduce here a multi-attribute, multi-expert model where several attributes are used for evaluating different e-learning paths, according to the rankings expressed by a group of experts. Then, a consensus modelling mechanism is introduced to find an agreement among the individual rankings.

The multi-attribute evaluation is based on fuzzy TOPSIS while the consensual ranking is obtained through a constrained optimization model. Fuzzy logic in e-learning has been used according to different perspectives. Some fuzzy approaches to e-learning have been presented in [3], where fuzzy logic has been

applied to the identification of e-learning design requirements and to select the most suitable e-learning service provider. Other approaches [4] use fuzzy inference to analyze students' way of working and group's behavior, while in other research areas fuzzy logic has been used to improve search capabilities of Learning Management Systems (LMSs) [5]. In the field of evaluation, under different perspectives we find the application of fuzzy logic to the evaluation of students' performances according to their profile [6], or to an evaluation teaching systems' quality [7].

2. The evaluation method based on fuzzy TOPSIS

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method is a popular approach to multi-attribute decision making problems. It was first developed by Hwang and Yoon [8]. Assuming that there are N alternatives and M attributes, the procedure of TOPSIS starts from the construction of the scores matrix $X=[x_{ij}]$ where x_{ij} denotes score of the i th alternative with respect to the j th attribute, and can be summarized as follows:

Step 1: Calculation of normalized decision matrix $Z=[z_{ij}]$

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}}, j = 1, \dots, m, i = 1, \dots, n \quad (1)$$

Step 2: Calculation of the weighted normalized decision matrix $V=[v_{ij}]$

$$v_{ij} = z_{ij}(\cdot)w_j, j = 1, \dots, m, i = 1, \dots, n \quad (2)$$

Step 3: Determination of the positive and negative ideal solution A^+ and A^- :

$$A^+ = \{v_1^+, \dots, v_n^+\} = \{(max_j v_{ij} | i \in B), (min_j v_{ij} | i \in C)\} \quad (3)$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \{(min_j v_{ij} | i \in B), (max_j v_{ij} | i \in C)\} \quad (4)$$

Where B is for benefit attributes and C is for cost attributes.

In educational settings, "benefits" can be interpreted as attributes contributing to maximize the effectiveness of learning objectives, while "costs" will be interpreted as attributes that could contribute to invalidate or nullify learning objectives of the educational paths.

Step 4: Calculation of the distance of each alternative from the positive ideal solution and negative ideal solution:

$$d_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^+)^2}, i=1, \dots, n \quad (5)$$

$$d_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2}, i=1, \dots, n \quad (6)$$

Step 5: Calculation of the relative closeness coefficients to the ideal solutions:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1, \dots, n \quad (7)$$

Step 6: Ranking of alternatives: The closer the CC_i is to one implies the higher priority of the i th alternative.

The formulation of TOPSIS, so far introduced, is based on a numerical representation of scores, and we know that it's not the most suitable to capture the uncertainty and imprecision inherent in the judgments expressed by experts when attributes are of qualitative nature. In this case, the ratings of qualitative criteria are considered as linguistic variables whose values can be represented as positive triangular fuzzy numbers (for the extension of TOPSIS to a fuzzy environment see [9] [10] [11]).

Here a triangular fuzzy number is defined as :

$$\begin{cases} 1 - \frac{a-x}{\alpha} & \text{if } a - \alpha \leq x \leq a \\ 1 - \frac{x-a}{\beta} & \text{if } a \leq x \leq a + \beta \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and denoted with (a, α, β) . The triangular fuzzy number is positive when the extremes of every α -cut are positive.

We assume now that are given respectively the set of experts $E=\{e_1, \dots, e_K\}$, a set of N e-learning paths (alternatives), and the set $L=\{l_1, \dots, l_S\}$ of linguistic terms used by the experts to estimate the TOPSIS matrices, denoted with $\bar{X}^{(1)}, \dots, \bar{X}^{(K)}$.

The elements of L and the corresponding positive triangular fuzzy numbers are here chosen e.g. as:

Very Low (VL)	(0, 0, 1)
Low (L)	(1, 1, 2)
Medium Low (ML)	(3, 2, 2)
Fair (F)	(5, 2, 2)
Medium High (MH)	(7, 2, 2)
High (H)	(9, 2, 1)
Very High (VH)	(10, 1, 0)

The algorithm of the multi-attribute multi-expert fuzzy TOPSIS-based evaluation can be summarized as follows:

Step 1: Construct the fuzzy matrix of scores for each expert using the linguistic terms in L and the corresponding positive fuzzy numbers previously introduced.

Step 2: Construct the normalized fuzzy matrices in such a way to preserve the property that the ranges of normalized triangular fuzzy numbers belong to $[0, 1]$.

Step 3: Introduce the weights of attributes (each expert has her/his own weights which are

represented by positive triangular fuzzy numbers) and then construct the weighted normalized fuzzy matrices.

Step 4: Determine the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

Step 5: Calculate the distance of each component from FPIS and FNIS, respectively, where the distance between two triangular fuzzy numbers, as chosen from the family of distances defined in [12], is assumed to be

$$D(\mathbf{x}, \mathbf{y}) = (I_L + I_R) / 2 \quad (9)$$

where $\mathbf{x} = (x, \alpha, \beta)$ and $\mathbf{y} = (y, \gamma, \delta)$ are triangular fuzzy numbers,

$$I_L = \int_0^1 (x_L(t) - y_L(t))^2 dt,$$

$$I_R = \int_0^1 (x_R(t) - y_R(t))^2 dt, \text{ and}$$

$[x_L(t), x_R(t)] = [x - \alpha + \alpha t, x + \beta - \beta t]$, $[y_L(t), y_R(t)] = [y - \gamma + \gamma t, y + \delta - \delta t]$ are the alpha-cuts of \mathbf{x} and \mathbf{y} respectively.

Step 6: Calculate the closeness coefficient for each component.

Step 7: According to the closeness coefficients, the ranking orders of all components is determined for every expert.

Step 8: Determine the ranking order that can be considered the best compromise (consensual ranking) for the group of experts.

While the first 7 steps can be implemented following, e.g., the approach developed in [10], to carry on the last step a consensus driving mechanism is introduced, taking into account the individual rankings and assuming that weights are introduced to measure the relative importance of experts. Suppose now that the ranking vector of expert k , $k=1, \dots, K$, as determined in Step 7, is represented by $R^k = (r_1^k, \dots, r_N^k)$, where r_i^k is the rank given to the j th path. Denote then with $R^G = (r_1^G, \dots, r_N^G)$ the group aggregated ranking. Assuming that the relative importance of the experts is measured introducing the weighting vector $w = (w_1, \dots, w_K)$, with $w_k \geq 0$, $k=1, \dots, K$, and $\sum_k w_k = 1$,

now we introduce the following OWA-based group distance functions

$$D_{OWA}(R^1, \dots, R^K) = \sum_k w_k d(R^{\pi(k)}, R^G) \quad (10)$$

where $d(R^{\pi(k)}, R^G) \geq d(R^{\pi(k+1)}, R^G)$, $k=1, \dots, K-1$.

The OWA aggregation operator was introduced in [13] and its main characteristic is that it allocates weights according to the input value, thus emphasizing in our situation the largest, the smallest or mid-range individual disagreements (distances).

Hence, we address the problem of finding the group ranking that represents the ranking of the K experts as accurately as possible, i.e. the ranking that minimizes the disagreement between individuals. The problem is represented as the following optimization model

$\text{Min } D_{OWA}(R^1, \dots, R^K)$, s.t. $R^G \in R$, where R is the subset of feasible rankings in P .

This kind of approach belongs to the class of distance-based models for aggregating ordinal preferences on a set of alternatives into a consensual ranking (see, among others, [14] [15] [16] [17]).

In order to find a compromise solution we assume that the metric d is the so called Spearman foot rule distance, and that the distances between individual rankings and the group ranking cannot overcome a given threshold. Accordingly, the optimization model becomes

$$\begin{aligned} \text{Min}_{R^k} \sum_{k=1}^K w_k \sum_{i=1}^N |r_i^{\pi(k)} - r_i^G| \quad (11) \\ \text{s.t. } \sum_{i=1}^N |r_i^{\pi(k)} - r_i^G| \leq \tau_k, k=1, \dots, K \end{aligned}$$

3. A context for the application of the model

Nowadays, resources wasting prevention is a must for every public administration, and the digitization of processes in order to replace (among the others) traditional paper-based procedures is an opportunity to contribute to this prevention. In this field, the term “dematerialization” has been used to identify the progressive elimination of paper-based processes in favor of their digitization. There is a strong commitment inside public administrations towards dematerialization, and this is the point where our use case wants to intervene through e-learning. Among the many novelties, four are particularly relevant:

- the use of electronic signature for signing digital documents
- the use of certified emails
- the use of digital protocol to track in/out movements of documents
- rules about digital preservation of electronic documents.

These four elements are clearly revolutionizing the Italian public administrations' processes, allowing new scenarios for the interaction between PAs (G2G), PAs and citizens (G2C), and PAs and companies (G2B).

Several e-learning paths and learning objects with different coverage of the above topics have been created by many public and private organizations, and a lot of people inside companies and public

administrations in Italy are involved in this training. In a situation where an educational institution should evaluate different proposals for educational paths on this topic, the attributes to be introduced in the model could be the following:

- clarity of language
- completeness
- adequacy of literature
- length of learning objects
- length of learning path
- structure of educational paths
- appropriateness of LO
- appropriateness of evaluation methods

The same mechanism and the same attributes, or variations of them, can be applied to a different granularity of objects inside a LMS. For example, very frequently in e-learning settings a teacher can use collaborative tools like forums or wikis to discuss over a topic. The comments of the users are often summarized or even pointed as “the best”, the most representative response to the original post even coming from participants in form of a question [18].

The model we are providing could be applied also to these contexts, where a panel of experts (teachers, students or a mix of them) could evaluate the different alternatives (the different answers to a question) using linguistic values selected from a given vocabulary.

4. Conclusions

In this paper, a method for supporting the evaluation of e-learning paths by a group of experts has been introduced. The system uses a two-stage decision making process, where in the first stage each expert evaluates alternative paths using a TOPSIS-based approach, assuming that the scores are linguistically expressed. The computations of individual rankings are carried out representing linguistic labels as positive triangular fuzzy numbers. The second stage is devoted to the description of the consensus modelling process aiming at finding the group ranking according to the minimization of a distance function.

The most remarkable novelty of our approach consists in proposing a mixed procedure which permits to combine individual ranking of e-learning paths, as carried in a multi-attribute setting by each member of a group of experts, with a linear constrained optimization process whose purpose is to determine a distance-based group consensual ranking.

This approach could be applied to the testbed we proposed in this paper, i.e., evaluation of different e-learning path proposals, but could also be extended to other settings inside learning environments, where multi-attribute and multi-expert evaluation can be applied. Examples of these application fields are those situations where a reputation attribute must be derived from the evaluation of an experts' panel respect to the contribution of different learners.

The voting mechanism in a forum, the selection of a wiki item's proposal respect to different proposals made by learners, or the item added to the FAQs by different contributors and evaluated by a team of experts

(teachers or simply other participants to the learning community) are examples of the application of our model to e-learning settings.

5. References

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