

Figure 6: Associative, continuous, idempotent aggregation functions on  $[0, 1]^2$ .

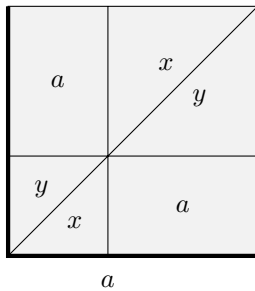


Figure 7: Associative, continuous, idempotent, commutative aggregation functions on  $[0, 1]^2$ .

The two cases  $a < b$  and  $a > b$  can be seen in Fig. 6.

**Remark 4.5** A continuous associative aggregation function  $F$  is commutative if and only if  $a = b$ . In this case the aggregation functions, shown in Fig. 7, are the following:

$$F(x, y) = \begin{cases} x \vee y & \text{if } x, y \in [0, \alpha]^2 \\ x \wedge y & \text{if } x, y \in [\beta, 1]^2 \\ a & \text{otherwise.} \end{cases}$$

## 5. Conclusions

Firstly, we have described how a binary operation on the unit interval  $[0, 1]$  should be in order to transform two implication functions in another implication function.

We have considered some aspects of the aggregated implication functions in terms of the two given ones and also we have analysed under which conditions the aggregation operation preserves some significant properties of the involved implication functions.

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## References

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