Book Reviews by F Calogero

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Seven books published by Birkhäuser are reviewed.

Garth Baker and **Alexandre Freire**, editors: *Nonlinear Partial Differential Equations in Geometry and Physics - The 1955 Barrett Lectures.* Birkhäuser, Basel Boston Berlin, 1997. 168 pages (DM 68,00).

V I Arnold, I M Gelfand, M Smirnov and **V S Retakh**, editors: *Arnold-Gelfand Mathematical Seminars - Geometry and Singularity Theory.* Birkhäuser. Boston Basel Berlin, 1997. 445 pages (DM 178,00).

Here we review two collections of essays on nonlinear differential equations, geometry and physics.

The first book consists of the Proceedings of 4 sets of Barrett Lectures presented in March 1995 at the University of Tennesse in Knoxville, as given by Ronald Fintushel ("*New Directions in 4-Manifold Theory*"), Sergiu Klainerman ("*On the Regularity of Classical Field Theories in Minkowsky Space-time R^{3+1}*"), Fang-Hua Lin ("*Static and Moving Vortices in Ginzburg-Landau Theories*") and Michael Struwe ("*Wave Maps*"). The preface notes that these lectures deal with "*topics which represent some of the most active areas of research today in the field of partial differential equations originating from problems in geometry, topology or theoretical physics. The lectures are of an expository nature, and directed at graduate students and researchers in the interface of geometric analysis and mathematical physics.*" In fact, the level of expertise required to appreciate these texts is rather high.

The (unsigned) Preface to the second book states: "*It is very tempting but a little bit dangerous to compare the style of two great mathematicians or of their schools. I think that it would be better to compare papers from both schools dedicated to one area, geometry and to leave conclusions to a reader of this volume.*" The book contains 19 research papers in English and one in French, arranged in alphabetical order (last name of first author); most of them appear to have been written specifically for this book in 1995-1996; one is authored by V Arnold ("*Topological Classification of Real Trigonometric Polynomials and Cyclic Serpents Polyhedron*"), three are co-authored by I M Gelfand (see below); in the Preface, it is noted that the "*papers from Arnold's seminar are devoted to three important directions developed by his school: Sympletic Geometry (F Lalonde and D McDuff), Theory of singularities and its applications (F Aicardi, I Bogaevski, M Kazarian), Geometry of Curves and Manifolds (S Anisov, V Chekanov, L. Guieu, E Mourre and V Ovsienko, S Gusein-Zade and S Natanzon). ... Papers from Gelfand's seminar are complementary to a recent Gelfand seminar volume and are more or less related by the notion of integral trasforms. An application of Radon transforms to a solution of Hilbert's fourth problem is*

given by J C Alvarez, I Gelfand and M Smirnov, nonlinear integrable equations and non-linear Fourier transform are considered by A Fokas, I Gelfand and M Zyzkin. Two papers (I Gelfand, M Graev and A Postnikov, A Kazarnovski-Krol) are devoted to hypergeometric functions..."

My opinion on such compilations tended to be generally negative: if the contributions printed there deserve to appear as scientific papers in refereed journals, they better be published as such; otherwise, they should not be published at all. This extremely intolerant view had some justification in the pre-INTERNET era, when the (worthy) papers published in such compilations were *de facto* only available to the happy few having access to good libraries. However, some of the contributions in such compilations have a size which makes them unsuitable for most scientific journals; for instance the first paper of this book (Francesca Aicardi: *"Discriminants and local invariants of planar fronts"*) fills 76 pages, and even the very interesting paper by Fokas, Gelfand and Zyskin (*"Nonlinear integrable equations and nonlinear Fourier transform)"* - the only one I feel quite qualified to understand and appreciate - with its over 30 pages and only 5 references - might have met some difficulties in seeking publication elsewhere. I am in any case convinced that most researchers working on nonlinear integrable systems would be quite happy to possess this book; a privilege, I am afraid, few will enjoy, due to its cost, which however does not appear altogether outlandish - for libraries - given its substantive (material, and especially scholarly) bulk.

Tiberiu Constantinescu: *Schur Parameters, Factorization and Dilation Problems*. Birkhäuser, Basel Boston Berlin, 1996. 264 pages (DM 148,00).

Yuri Egorov and **Vladimir Kondratiev**: *On Spectral Theory of Elliptic Operators*. Birkhäuser, Basel Boston Berlin, 1996. 338 pages (SFR 158,00).

The two books reviewed here are the items no. 82 and no. 89 of the series on *Operator Theory: Advances and Applications*, edited by I. Gohberg and supervised by an impressive Editorial Board featuring 39 other distinguished mathematicians, as well as by an even more impressive Honorary and Advisory Editorial Board featuring 6 superstars: P R Halmos, T Kato, P D Lax, M S Livshic, R Phillips and B Sz -Nagy.

"This book is devoted to the ubiquity of the Schur parameters. A dilation theoretic view leads to a unified perspective on several topics where Schur parameters appear as basic cells. Together with the transmission line, their physical counterpart, they appear in scattering theory, in modeling, prediction and filtering of nonstationary processes, in signal processing, geophysics and system theory. Modelling problems are considered for certain classes of operators, interpolation problems, determinantal formulae, as well as connections with certain classes of graphs where, again, the Schur parameters could play a role. Some geeral algorithms that explore the transmission line are also presented in this book. As a whole, the text is self-contained and it is addressed to people interested in the previously mentioned topics or connections between them." Thus reads the cover blurb of the first book. While it would be difficult to improve on this brief yet complete description, a word of warning should be voiced: those primarily interested in some of these applications will have considerable difficulty in extracting useful results from this

book, unless they are prepared to study it carefully from the beginning (this is not the book an engineer might open and use at a glance).

Every one of the 8 Chapters is followed by a terse Section devoted to bibliographical notes. References are collected at the end, and take 28 pages; and there is a 3-page Index. The price suggests that the targeted customers are libraries rather than individuals.

The second book reports results obtained by its two authors in 1980-1992, which were previously published only partially, some of them only in Russian. These results provide estimates of eigenvalues, especially of the first one, and of eigenfunctions of elliptic operators, including Schroedinger equations (in spaces with various numbers of dimensions). The main tools used are the variational principle and *a priori* estimates, generally in Sobolev spaces. The first part of the book (Chapters 1-4 and Section 5.1) reviews the basic theory of Hilbert and Sobolev spaces and of differential equations, at a level suitable for "*senior University students.*" Chapters 5-8 are devoted to *The Sturm-Liouville Problem, Differential Operators of Any Order, Eigenfunctions of Elliptic Operators in Bounded Domains*, and the *Negative Spectra of Elliptic Operators.* There is a 7-page Bibliography (in alphabetical order), and a 2-page Index.

H Dym, B Fritzsche, V Katsnelson and **B Kirstein**, editors: *Topics in Interpolation Theory.* Birkhäuser, Basel Boston Berlin, 1997. 516 pages (DM 188,00).

This book is largely based on the meeting "Recent Developments in Schur Analysis, A Workshop in Honour of the 80th Bithday of V P Potapov" held at Leipzig University in August 1994. Vladimir Petrovich Potapov was a mathematician who worked mostly in Odessa; he moved to Kharkov in May 1976 and died there in December 1980. He was a very original man, as it is clear form the reminiscences of his students and friends presented as a chapter in this book: particularly noteworthly for his sense of humor is the contribution (8 pages) by Moshe S Livshic, who was one year older than Vladimir Petrovich and a close friend of his since their student days in Odessa. The other contributors to this chapter are D Z Arov, L A Sakhnovich, A A Nudelman, V L Dubovoj and V E Katsnelson.

The book contains in addition 22 scientific papers, ordered alphabetically (last name of first author); about half of them are based on contributions presented at the meeting, the other ones include the translations of "*a number of important papers (which serve to clarify the Potapov approach to problems of interpolation and extension, as well as a number of related problems and methods) and are relatively unknown in the West*" and "*two expository papers, which have been especially written for this volume.*"

A terse summary of the topics treated, and the authors of the relevant contributions, can be extracted from the *Editorial introduction* in the following guise: Multiplicative decompositions (Yu P Ginzburg; M S Livshic, I V Mikhailova; V I Smirnov), Fundamental matrix inequalities (V K Dubovoj; Yu M Dyukarev; L B Golinskii and I V Mikhailova; V E Katsnelson; I V Kovalishina), Canonical systems of differential equations (M G Krein; I V Mikhailova and V P Potapov; L A Sakhnovich), The abstract interpolation problem (V E Katsnelson, A Ya Kheifets and P M Yuditskii; H Dym and B Freydin), Spaces with an indefinite metric (N I Akhiezer; A Dijksma and H Langer; E Russakovskii), Other directions (D Alpay and V Bolotnikov; L B Golinskii; V E Katsnelson and B Kirstein; S Kupin and P M Yuditskii).

Wolfang Hackbusch: *Integral Equations - Theory and Numerical Treatment.* Birkhäuser, Basel Boston Berlin, 1995, 376 Pages (DM 98,00).

Ram P Kanwal: *Linear Integral Equations - Theory & Technique.* Birkhäuser, Boston Berlin, 1997, 328 pages (DM 148,00).

We review here two recent books on integral equations.

The first book, based on lectures given to students of mathematics, is a translated and revised version, done by the author himself, of a book published in German in 1989. Readers are expected to possess the standard background in calculus and numerical analysis, and a minimum of functional analysis. But the book reviews at the beginning, briefly but very clearly, all the higher mathematics beyond basic calculus used in later chapters. The presentation is compact but easy to follow, rigorous but not tediously so; the coverage is quite complete, including singular and complex equations besides the standard material (Volterra, Fredholm). Much emphasis is given to numerical techniques, including examples of specific computations; this is indeed the special field of expertise of the author. There is a Bibliography (covering 9 pages), and an Index (7 pages). The price, given current standards, seems reasonable - although a cheaper paperback edition would be necessary for students.

The second book is an updated version of a book published in 1971 by Academic Press. The presentation is considerably more elementary than that of the book reviewed above. The coverage includes, in addition to the standard material (Volterra, Fredholm, Abel, Hilbert), the treatment of boundary value problems and perturbation theory, including various applications; it is of course restricted to linear equations, as indicated by its title. There is an Index (5 pages), and a Bibliography (3 pages; but the claim, made in the Preface to this Second Edition, that it has been brought up to date, seems hardly warranted - except for the inclusion of papers by the author of the book). The price seems excessive.

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