Annex

Hereafter, we present some calculus elements for the determination of the distribution function of the sum of two identical independent symmetric Pareto distributions:

$p_{SP}(x) = \frac{1}{2(1+|x|)^2}$ gives by integration :

$$F_{SP}(x) = \frac{1}{2(1-x)}, x \leq 0;$$

$$F_{SP}(x) = 1 - \frac{1}{2(1+x)}, x \geq 0$$

The expression of the distribution function of the sum Y for $x \geq 0$ is:

$$F_{SP}(x) = \frac{1}{2(1-x)}, x \leq 0;$$

$$F_{SP}(x) = 1 - \frac{1}{2(1+x)}, x \geq 0$$

The expression of the distribution function of the sum Y for $x \geq 0$ is:

$$G_Y(y) = \int_{-\infty}^{0} F_{SP}(y-x)\, p_{SP}(x)dx$$

$$+\int_{0}^{x} F_{SP}(y-x)\, p_{SP}(x)dx + \int_{x}^{\infty} F_{SP}(y-x)\, p_{SP}(x)dx$$

Further, each of the three integrals is computed by making a decomposition in simple elements that leads to the following result:

$$G(y) = \frac{1}{2} + \frac{1}{2}.\frac{\ln(1+y)}{y^2} - \frac{1}{4}.\frac{(2+y)}{y.(1+y)} + \frac{1}{2}\frac{y}{1+y}$$

$$- \frac{1}{2}.\frac{\ln(1+y)}{3+2y+y^2} - \frac{1}{4}\frac{y.(2+y)}{(1+y).(3+2y+y^2)}$$