Appendix A

$$X_{E_I^n} = \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + (\mu_{I_E}+nd_u)x_I}{\sum_{i=1}^{M}\mu_{i_E}+nd_u} = \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + \mu_{I_E}x_I + nd_ux_I}{\|E\|+nd_u}.$$

$$\begin{aligned} X_{E_I^{n-1}} &= \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + (\mu_{I_E}+(n-1)d_u)x_I}{\sum_{i=1}^{M}\mu_{i_E}+(n-1)d_u} \\ &= \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + \mu_{I_E}x_I + nd_ux_I - d_ux_I}{\|E\|+(n-1)d_u}. \end{aligned}$$

Therefore

$$\begin{aligned} X_{E_I^n} - X_{E_I^{n-1}} &= \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + \mu_{I_E}x_I + nd_ux_I}{\|E\|+nd_u} \\ &\quad - \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i + \sum_{i=I+1}^{M}\mu_{i_E}x_i + \mu_{I_E}x_I + nd_ux_I - d_ux_I}{\|E\|+(n-1)d_u} \end{aligned}$$

$$\begin{aligned} &= \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i(n-1)d_u + \sum_{i=I+1}^{M}\mu_{i_E}x_i(n-1)d_u + \mu_{I_E}x_I(n-1)d_u}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)} \\ &\quad + \frac{nd_ux_I(n-1)d_u - \sum_{i=1}^{I-1}\mu_{i_E}x_ind_u - \sum_{i=I+1}^{M}\mu_{i_E}x_ind_u}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)} \\ &\quad + \frac{-\mu_{I_E}x_Ind_u - x_In^2(d_u)^2 + d_ux_I\|E\| + x_In(d_u)^2}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)}. \end{aligned}$$

Since

$$\sum_{i=1}^{M}\mu_{i_E}x_ind_u = \sum_{i=1}^{I-1}\mu_{i_E}x_ind_u + \sum_{i=I+1}^{M}\mu_{i_E}x_ind_u + \mu_{I_E}x_Ind_u,$$

the numerator may be simplified to obtain:

$$\begin{aligned} X_{E_I^n} - X_{E_I^{n-1}} &= \frac{\sum_{i=1}^{I-1}\mu_{i_E}x_i(n-1)d_u + \sum_{i=I+1}^{M}\mu_{i_E}x_i(n-1)d_u + \mu_{I_E}x_I(n-1)d_u}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)} \\ &\quad + \frac{nd_ux_I(n-1)d_u - \sum_{i=1}^{M}\mu_{i_E}x_ind_u - x_In^2(d_u)^2 + d_ux_I\|E\| + x_In(d_u)^2}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)}. \end{aligned}$$

Moreover, since

$$\sum_{i=1}^{M}\mu_{i_E}x_i(n-1)d_u = \sum_{i=1}^{I-1}\mu_{i_E}x_i(n-1)d_u + \sum_{i=I+1}^{M}\mu_{i_E}x_i(n-1)d_u + \mu_{I_E}x_I(n-1)d_u,$$

the numerator may be further simplified to give:

$$X_{E_I^n} - X_{E_I^{n-1}} = \frac{d_ux_I\|E\| - \sum_{i=1}^{M}\mu_{i_E}x_id_u}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)}.$$

We know that

$$\sum_{i=1}^{M}\mu_{i_E}x_i = X_E\|E\|.$$

It follows that

$$X_{E_I^n} - X_{E_I^{n-1}} = \frac{d_ux_I\|E\| - X_E\|E\|d_u}{(\|E\|+nd_u)(v+(n-1)d_u)} = \frac{\|E\|d_u(x_I - X_E)}{(\|E\|+nd_u)(\|E\|+(n-1)d_u)}.$$