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Reflections on the teaching of Fuzzy Logic

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Abstract

In this paper, it is done a reflection on the importance of popularizing Fuzzy Logic in Basic Education. Some clues for introducing few basic ideas in schools and high schools are shown. The relations between Fuzzy Logic and many different fields are also indicated.

Keywords: Basic Education, Compositional Rule of Inference, Fuzzy Logic, Fuzzy Set, Teaching.

1. Introduction

This paper is devoted to collect questions about the role of Fuzzy Logic along the educational process. Nowadays, this topic is, at most, included in the program of some University degrees or in post-degrees courses. Then, the popular believe, is that Fuzzy Logic is a complex topic that used sophisticate tools. This is the main idea I would want to break in this paper.

It is obvious that the relevance of Fuzzy Logic comes with technological developments ([1], [2]), but the essence of Fuzzy Logic goes further of its applications. In 1965 Zadeh introduced the concept of a Fuzzy Set [3], this fact enlarges the definition of a Classical Set, it allows the existence of elements that are not necessary in the set or outside, elements can belong to the set with a degree.

Contemporary to the first paper of Fuzzy Sets, the Bourbaki group (Association des collaborateurs de Nicolas Bourbaki), proposed the construction of all Mathematical Theory based on Theory of Sets ([4]). So, this showed the importance of Set Theory (in classical sense). Many followers of that idea encourage changing the way of teaching Maths, even in schools, strengthening the idea that Mathematics contribute to from logic structures in the mind what allow people to reach logical consequences. In that point, Maths stopped being considered as a tool that provides calculating algorithms. The path carrying out for this aim is the popularization of set theories in basic education, under the name of Modern or Abstract Mathematics. Once setting the axioms, Mathematical knowledge could be built. This fact, goes together with the Piaget's Constructivism [5].

North Americans bet for a change on the classical education of Mathematics, and adopted the ideas of Modern Mathematics. On other hand, it seems that this change brought also problems in the ways of learning, this Mathematics are not intuitive and are even consider out of the real world scope.

Following the ideas of Constructivism and the relation with intuition, Fuzzy Set Theory could appear naturally, since its aim is to formalize people intuitions and the theory is built over strong but, under a mathematical point of view, not very difficult bases.

2. Developing a new Science: the popularization

Under my view, a crucial steep in the developing of a new Science is the popularization, which is usually a controversial issue, since not all scientific progresses are naturally shown to the population. How to decide which development should be shown?

I strongly consider that Fuzzy Logic is a complete discipline that embraces the classical conception of Maths (see figure 1) but not only. This field deals with the imprecision, present in our live and classically given up by Maths. So, introducing something that does not follow classical schemes is even more hard than introducing little changes. In that direction, we should thanks Statistics and Probability, one of the youngest branches of Mathematics that are, nowadays, spreading along the educational systems and that are proven that are needed in many scientific disciplines but also in humanities.

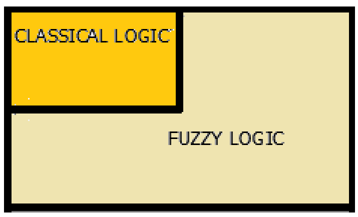

Figure 1: Fuzzy Logic vs Classical Logic

Anyway, in the beginnings of Probability, many mathematicians are against including it in the

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formal frame of Maths because its worth, rigor and thoroughness. The first item in the path of its acceptance, was Kolmogorov axiomatic [6], but then many authors among them are marked, L. Santaló [7] and J. Franklin [8] contributed to the development and acceptance of that field.

I would like to remark that Probability is used to represent uncertainty and Fuzzy Sets imprecision. Both terms model unknown aspects, but they model two kinds of ignorance. Uncertainty is the one that goes together with an event before its realization and it is based on some background knowledge of the event, usually information on the results from the experience of previous realizations, and after the realization of the experience the uncertainty disappears. In the case of uncertainty, probability is encourage to model those situations.

On the other hand, imprecision, is not a matter of modelling experiences, or physical phenomena, it is related to humans activities. For instance, in language, when a word is stated in a concrete language all people knowing that language understand the meaning, although it is not precise because it depends of the use, the context and the purpose. Following that idea a model for the meaning of predicates will be shown using fuzzy sets.

Fuzzy Logic is different of probability [9]. In fact, in order to define a probability it is necessary a boolean structure and that is not the case of Fuzzy Logic, so, Fuzzy Logic is not the same, and under my view, it can not be considered a kind of extension of the model of probability for represent unknown events.

Notwithstanding, both topics are new branches of Maths that could be present in many different disciplines.

2.1. Fuzzy Logic in different fields

Fuzzy Logic could be useful in many disciplines because of its capacity of collecting the imprecision in language, for instance the imprecision of many terms used in Psychology, Social Sciences, Philosophy, it should not be a exclusive tool of science or engineering problems. I argue the proposal of introducing Fuzzy Logic in many fields such as:

- **Psychology**. It is usually to deal with imprecise concepts which could be modelled using fuzzy sets, allocating different degrees and getting closer information for each individual. The mentioned imprecise concepts are for instance: Gifted children, attention deficit disorder, creativity, egocentrism, shyness, obsessive behavior, sociability, ways of learning,...
- **SocialSciences**. There are many measures useful in Social Sciences, that could be measure with fuzzy sets. For instance: birth rate, death rate, unemployment rate, education levels of population,...
- **Philosophy**. Along the history Philosophy has been encouraged the study of thinking, and also the study of logic, since Fuzzy Logic is a new concept of logic, it must be studied in philosophical terms.
- **Language**. Fuzzy Logic could model, at least, some gradable predicates and sentences (conditionals), that is why linguistics should be interested in the aspect of Fuzzy Logic regarding the way of collecting the use, the contest, and the purpose or words in mathematical terms.

On the other hand, the basic mathematical elements that hold Fuzzy Logic, allow this theory to be included in basic scope, not only in University. What is more, under my view, it could be also shown in schools.

In order to put together the experience of pupils and mathematical models, this paper claims the importance of teaching Fuzzy Logic, and it is proposed to start teaching its main ideas (imprecision, relation with language,..) from the first stage of child's education.

To explain the concept of length, in the Montessori's learning method, children learn this concept by ordering several sticks of different sizes. So, following a similar methodology they could learn a concept like *big*, which is a fuzzy concept that could be represented by a fuzzy set.

Why nowadays Mathematics Education tries to avoid imprecision? Without taking imprecision into account logic is abstract and it is a formalism far from reality. Maybe this fact contributes to make Mathematics something dark, complex and involving too many difficulties in its study.

Although I consider introducing Fuzzy Theory in Basic Education could contribute to change the common believe that Mathematics are far from reality, I am aware that children will have many problems to break their dichotomic world (for them things are good or bad). Notwithstanding, the fact of breaking this dichotomy comes with the maturation process.

3. Motivation

There is no doubt that the new field of Fuzzy Logic or Soft Computing or even, Computing with Words, has acquired a huge importance in the last half century, it is used in many real world problems and good results are obtained. In addition, this theory is based on the intuition of people materialized in the design of each fuzzy set [10]. So, Fuzzy Logic collects the practical experience which is desirable for constructivism.

In order to encourage society and teaching community to introduce some aspect of Fuzzy Logic in the basic educational programs, some contributions of this logic, under a theoretical and a technological point of view, are remarked.

3.1. Theoretical point of view

The main idea of Fuzzy Logic is to introduce propositions that are not necessarily true of false, what makes a logic closer to reality, capable of translate humans perceptions and languages. Therefore, it could be considered to be more related to humans intuition than Classical Logic, having the same solid mathematical bases.

On the contrary, the flexibility of Fuzzy Logic, force to frame it in mathematical structures that are no as rigid as it is the case of Boolean Algebras. The strongest structure to work with in Theories of Fuzzy Set is a De Morgan Algebra. And usually, this is not so common, in fact, it is frequently to deal with partial orders or pre-orders (taking into account the pointwise order).

This aspect makes difficult to consider the verification of many classical laws, therefore from a theoretical point of view, Fuzzy Logic should be developed in the design of proper connectives in order to get the verification of classical laws. Following that idea, we have done some contributions, for instance: Reduction to absurd [11], weakening the antecedent or straightening the consequent in a conditional clause [12], properties of symmetric difference [13],...

3.2. Technological point of view

The importance of Fuzzy Logic is consolidated with many applications. Sugeno tackled intelligent control of an helicopter, and also industry processes as a wastewater treatment plant. In fact, many common electronic advises work with Fuzzy Logic, such as: washing machines, tensiometers, rice cookers,... and also cars capable of parking without a driver! All these machines, could be explained in order to motivate the study of Fuzzy Logic, since usually students need a visual and practical idea.

The theory under fuzzy control is not so complicate, so it could be explained the design of predicates and conditionals involved, and the Compositional Rule of Inference [14], and students could understand the behaviour of easy systems with two rules with a numerical consequent, or even more complex systems as the equilibrium of inverted pendulum (see figure 2) which could be controlled with Fuzzy Logic easier than with Classical Logic.

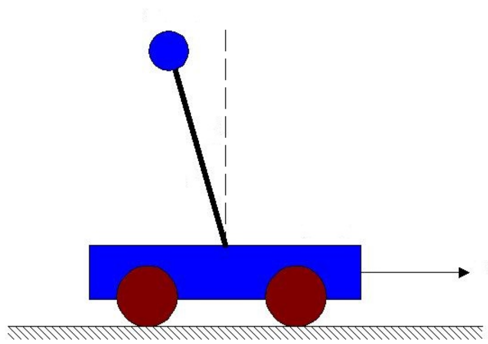

Figure 2: Inverted Pendulum

Apart from control problems, Fuzzy Logic is englobed in the term Soft Computing which includes many techniques with direct applications, such as: genetic algorithms, neuronal networks, collaborative intelligent systems, intelligent data analysis,...

4. Proposal

In this section I will shown a proposal of the contents of Fuzzy Logic along the educational system. Starting with the Education in the kindergarten, and continuing with schools and high schools in order to achieve any degree knowing some bases of Fuzzy Logic. It is possible that the most controversial idea is to introduce fuzzy concepts in the first years of a education process.

Figure 3 collects a graph showing the stages of education and the division in high schools in two blocks, one corresponding to studies in Sciences and the other to studies in Humanities.

4.1. Schools

During the school students are shown the bases of all their future education. Also bases of Logic, and why not bases of Fuzzy Logic? As it was mentioned Fuzzy Logic is more intuitive and has a stronger relation with the reality than Classical Logic. Therefore, students could assimilate it and the relation between logic (Mathematics) and reality becomes stronger than when dealing with Classical Logic.

I will divided this period in two different stages: the kindergarten and the primary school.

Regarding to the first stage of education, the idea of imprecision behind Fuzzy Logic could be introduced in some sense: introducing what a fuzzy set is.

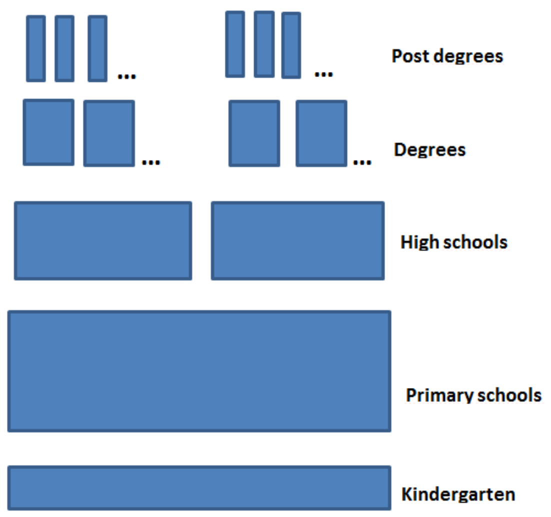

Figure 3: Educational Stages

- Pupils could be asked for selecting in a given universe of figures, the figures that verify a property, the figures that do not verify a property and the figures that partially verify the property.
- They could order themselves though their highs and be asked for what they consider is the collective of tall pupils.

These examples will allow them to understand the idea of the design of a fuzzy set since different pupils will consider different collectives. And, this is naturally linked with their curriculum in which they have to classify and order collections of things.

In this first stage of education, pupils need to structure they logic knowledge, understanding the meaning of intersection and union. At least, translating in language the intersection by *and* and the union by *or* and identify, for instance, the collective of tall *and* (*or*) with fair hair partners in their classes. In the same way they could be also asked to identify the collective of *non-tall* (or the opposite: *short*) pupils, introducing in a wide sense the negation. Introducing negation is a complicated matter since the non-reversible thinking of children in that period.

The first stages of mathematical education are also devoted to establish relations between elements. Nowadays, these kinds of relations are crisp, it is the case of relating two elements with the same color, but if dealing with not two equal colors, they could relate elements with the color they consider similar or in the same tone. Doing that flexible relations, the bases of fuzzy relations are established. Pupils could understand order and equivalence fuzzy relations, as, nowadays, they do with classical relations. In order to work with these relations some basic properties could be manipulative ([15]) show: reflexivity, a kind of transitivity, symmetric and anti-symmetric.

In Primary schools, they could continue with the previous concepts, but they could be enlarged with the representation of fuzzy sets. To represent a fuzzy set it is necessary to choose the Universe of discourse and allocate to each element in the universe a degree, between 0 and 1, of belonging to a fuzzy set, that idea could be formalized with representations. Starting with relevant examples for them, for instance, to represent the predicate tall taking the Universe of Discourse of their classmates, allocating a degree of being tall for each of them depending on their high. Later, they could do an analogous design with the the universe of discourse of the first fifteen natural numbers and the predicate *being around* 5. The idea of continuous fuzzy sets could be also introduced by interpolation (in previous example: extending the universe of discourse to the real numbers) (see Figure 4), notwithstanding continuity is a concept that will be better formalized in following courses.

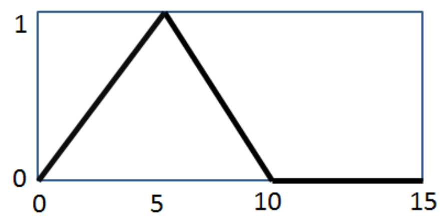

Figure 4: Fuzzy set: Around 5.

Relating with the concept of negation, students could understand the difference between negate a predicate and the antonym of a predicate. The negation could be built using the negation $N = 1 - id$ and represented in language by *no*. Following that idea they could represent the fuzzy set of *non-tall* classmates by the axial symmetry with respect to the line of the fixed point of the negation (0.5) and represent the antonym allocating a degree of each classmate of being *short*. They realize that the antonym of a predicate is a linguistic term. In High schools the existence of a mathematical construction of the antonym will be shown, for that aim it will be defined a symmetry in the universe of discourse instead of in the degree as it is the case of negations.

4.2. High schools

When arriving to High Schools, Students should understand what is a fuzzy set and how to built intersections, unions and negations, in a very simple way.

Enlarging their knowledge, it is possible to formalized those concepts using the characterization theorems of t-norms, t-conorms and strong negations.

4.2.1. Continuous t-norms

Intersections will be formalized by continuous t-norms, whose monotonic character in the variables is an intuitive property. In order to translate the meaning of intersection, the null element (0) is absorbent and the biggest element (1) is neutral. Then, continuity, commutativity and associativity are supposed in order to simplify the operativity. Although the particle *and* in language is not usually commutative.

So, a continuous t-norm ([16]), T, is a mapping $T : [0,1] \times [0,1] \to [0,1]$ verifying the following properties,

- Monotonicity $T(x, y) \leq T(z, t)$ if $x \leq z, y \leq t$
- $T(x, 1) = x, \ \forall x \in [0, 1]$
- $T(x, 0) = 0, \ \forall x \in [0, 1]$
- Associativity $T(T(x, y), z) = T(x, T(y, z))$, $\forall x, y, z \in [0, 1]$
- Commutativity $T(x, y) = T(y, x) \ \forall x \in [0, 1]$
- Continuity in both variables

Then, it could be shown as examples functions $T : [0, 1] \times [0, 1] \to [0, 1]$ such as,

- The function min.
- The product.
- The Łukasiewicz t-norm [1] $W(x, y) = max(0, x + y - 1)$.

Pupils should understand that for two different values, the t-norm allocates a third value, the value of their intersection. So, it could be seen as a three-dimensional graph. In the case of product, the result is shown in figure 5.

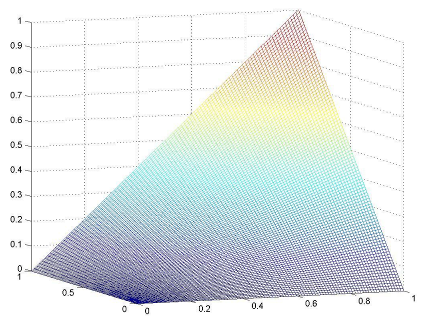

Figure 5: Product

Remark 4.1 *Many problems of design could be developed at class. As an example, students could do the intersection with two marks of the subject and calculating the resulting general mark with different t-norms. This allows to compare them and to obtain that the biggest is the minimum and the smallest the Łukasiewicz one, obtaining a mark of* 0 *if having two values less than the middle* (0.5)*. Even more, ordinal sum of t-norms could be introduced.*

[1] It could be remark that this t-norm has 0 divisors.

4.2.2. Continuous t-conorms

The scheme of introducing continuous t-conorms is equivalent to the scheme in 4.2.1. The properties of t-conorms that make the difference with respect to t-norms are that element 0 is neutral and element 1 is absorbent, something coherent if t-conorms try to translate the properties of the union.

So, a continuous t-conorm, S, is a mapping $S : [0, 1] \times [0, 1] \to [0, 1]$ verifying the following properties,

- Monotonicity $S(x, y) \leq S(z, t)$ if $x \leq z, y \leq t$
- $S(x, 0) = x, \ \forall x \in [0, 1]$
- $S(x, 1) = 0, \ \forall x \in [0, 1]$
- Associativity $S(S(x, y), z) = S(x, S(y, z)), \ \forall x, y, z \in [0, 1]$
- Commutativity $S(x, y) = S(y, x) \ \forall x \in [0, 1]$
- Continuity in both variables

Then, it could be shown as examples functions $S : [0, 1] \times [0, 1] \to [0, 1]$ such as,

- The function max.
- $Prod^*(x, y) = x + y - x \cdot y$.
- The Łukasiewicz t-conorm, $W^*(x, y) = min(1, x + y)$.

4.2.3. Negations

As it was mentioned, two different ways of negating a predicate will be shown. The first one is the negation of a fuzzy set, and the other is the opposite [17], obtained through a symmetry in the universe of discourse.

A negation, $N : [0, 1] \to [0, 1]$ is a function such that

1. If $a \leq b$, then $N(b) \leq N(a)$, for all a, b in L
2. $N(0) = 1$, and $N(1) = 0$.

and the negation of a fuzzy set μ is represented by $\mu' = N \circ \mu$.

Also, it could be shown that in order to translate the intuitive meaning of functionally expressible negations, it is suitable to count with

3. $N \circ N = \mathrm{id}_L$,

functions N verifying (1), (2), and (3) are called strong negations, and they are continuous.

As an example of negations, there could be introduced Sugeno's negations: $N_\lambda(a) = \frac{1-a}{1+\lambda a}$, with $\lambda > -1$, for all $a \in [0,1]$.

For example, $N_0(a) = 1 - a, N_1(a) = \frac{1-a}{1+a}, N_{-0.5} = \frac{1-a}{1-0.5a}, N_2(a) = \frac{1-a}{1+2a}$, etc.

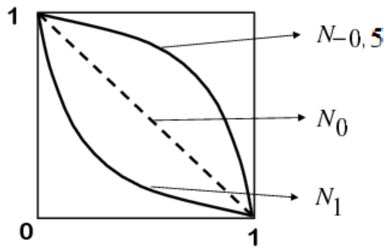

Figure 6: Sugeno's Negations

Since obviously,

$$N_{\lambda_1} \leq N_{\lambda_2} \Leftrightarrow \lambda_2 \leq \lambda_1,$$

it results:

- If $\lambda \in (-1, 0]$, then $N_0 \leq N_\lambda$
- If $\lambda \in (0, +\infty]$, then $N_\lambda < N_0$.

When using fuzzy sets to represent the meaning of predicates [18], the problem of design [10] is always present and this has to be told and shown in every concrete problem.

Many examples could appear when building the antonym of a predicate. For doing that, the symmetry has to be designed translating the meaning and also keeping the mathematical properties needed:

$A : X \to X$ is *symmetry* on X, if it is a function such that

- If $x \leq y$, then $A(y) \leq A(x)$
- $A \circ A = \mathrm{id}_X$,

Once $\mu_P : X \to [0,1]$ is known, take $\mu_{aP}(x) = \mu_P(A(x))$, for all x in X, that is, $\mu_{aP} = \mu_P \circ A$ represents an opposite of μ_P.

In order to motivate students for a deep study of Fuzzy Logic, some general ideas of meanings of predicates and some tools to solve engineering problems (control problems) could be shown. The problem of meaning is devoted to people who want to follow studies in Humanities and control problems are devoted to people who want to follow experimental studies, taking into account that the problem of meaning and design is also included in control problems.

4.3. Fuzzy control design

Once knowing algebras of fuzzy sets (T, S, N), some designs of particular t-norms in order to verify classical laws (such as Non Contradiction $T(\mu, N(\mu)) = 0$, Excluded Middle $S(\mu, N(\mu)) = 1$, De Morgan law,...) could be shown.

In the same vein, that is, continuing with the problem of design, students could be shown the amount of possible conditional functions existing (J) translating the sentence *If a, then b*, a conditional is a function verifying Modus Ponens $(T(J(a,b), a) \leq b)$, that is useful to do deductions present in any kind of forward reasoning. The most famous conditionals are:

- R-implications: $J_T(a,b) = sup\{z \in [0,1]; T(a,z) \leq b\}$
- S-implications: $J(a,b) = S(N(a), b)$
- Q-operators: $J(a,b) = S(N(a), T(a,b))$
- D-operators: $J(a,b) = S(b, T(N(a), N(b)))$
- ML-operators: $J(a,b) = T(a,b)$

Remark 4.2 *In the context of design it could be said that in the case of ML-operators the role of the consequent is the same as the antecedent, which not always could be desirable.*

Once these kind of conditionals are known, students could be show how to solve an easy control problem using rules. Fuzzy rules deal with imprecise terms and some imprecision is also allowed regarding the input of the system, it could be an approximation to the antecedent of the rule.

For solving those problems there are four steeps to follow:

- Design the predicates involved in the rules
- Design the conditional functions
- Apply the Compositional Rule of Inference (CRI) for obtaining the output μ_Q^* when having *If* μ_P*, then* μ_Q and the approximate input of μ_P^*.

$$\mu_{Q^*}(y) = \sup_{x \in [0,10]} \min(\mu_{P^*}(x), J_i(\mu_P(x), \mu_Q(y))).$$

- Calculate the general output $\mu_{Q^*} = \max_{i=1,\ldots,n}(\mu_{Q_i^*})$ in the case there are n rules.

Example 4.3 *For instance it is collected a simple example from [14].*

A system with two continuous variables $x, y \in [0,10]$*, described by 3 rules,*

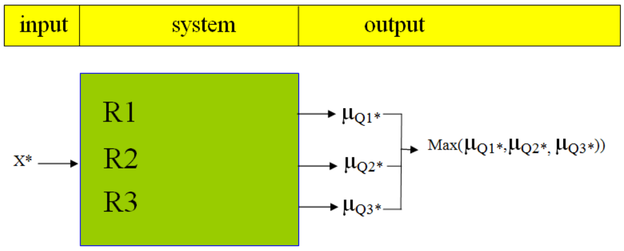

Figure 7: CRI scheme

r_1 : *If x is big, then $y = 2$*
r_2 : *If x is small, then $y = 8$*
r_3 : *If x is around 4, then $y = 6$.*

What is the output for the input x=4.5?

Firstly, predicates are represented:

To represent the sentence x is big, *it is possible to buil the fuzzy set,*

$$\mu_B : [0, 10] \to [0, 1]$$

$$x \mapsto \frac{x}{10}$$

To represent the sentence x is small,

$$\mu_S : [0, 10] \to [0, 1]$$

$$x \mapsto 1 - \frac{x}{10}$$

To represent the sentence x is around 4,

$$\mu_{A4} : [0, 10] \to [0, 1]$$

$$x \mapsto \begin{cases} 0, & \text{if } 0 \le x \le 3, 5 \le x \le 10 \\ x - 3, & \text{if } 3 \le x \le 4 \\ 5 - x, & \text{if } 4 \le x \le 5. \end{cases}$$

Then, it is necessary to represents rules. In this problem, let J_i be a Mamdani-Larsen implication $J_i(a, b) = T(a, b)$, and as it is usual in fuzzy systems control, we suppose that $J_i(a, b) = a{\cdot}b$. ($\forall i = 1, 2, 3$). Remember that doing that the implication if/then, *works as an* if and only if *implication.*

In the case of first rule, If x is big, then $y = 2$, *the representation is,*

$$J_1(\mu_B(x), \mu_{\{2\}}(y)) = \mu_B(x){\cdot}\mu_{\{2\}}(y) = \begin{cases} \frac{x}{10}, & \text{if } y = 2 \\ 0, & \text{if } y \ne 2. \end{cases}$$

In the case of the second rule r_2: If x is small, then $y = 8$,

$$J_2(\mu_S(x), \mu_{\{8\}}(y)) = \mu_S(x){\cdot}\mu_{\{8\}}(y) =$$

$$\begin{cases} 1 - \frac{x}{10}, & \text{if } y = 8 \\ 0, & \text{if } y \ne 8. \end{cases}$$

Regarding the third rule r_3: If x around 4, then $y = 6$,

$$J_3(\mu_{A4}(x), \mu_{\{6\}}(y)) = \mu_{A4}(x){\cdot}\mu_{\{6\}}(y) =$$

$$\begin{cases} \mu_{A4}(x), & \text{if } y = 6 \\ 0, & \text{if } y \ne 6. \end{cases}$$

Once having the conditionals and the input, the outputs are calculate using the conditional rule of inference, as follow:

- $\mu_{Q_i^*}(y) =$
  $= \sup_{x \in [0,10]} \min(\mu_{\{4.5\}}(x), J_i(\mu_{P_i}(x), \mu_{Q_i}(y))) =$
  $= J_i(\mu_{P_i}(4.5), \mu_{Q_i}(y))$

$\forall i = 1, 2, 3$

- $\mu_{Q_1^*}(y) = J_1(\mu_B(4.5), \mu_{\{2\}}(y)) = \begin{cases} 0.45, & \text{if } y = 2 \\ 0, & \text{if } y \ne 2, \end{cases}$
- $\mu_{Q_2^*}(y) = J_2(\mu_S(4.5), \mu_{\{8\}}(y)) = \begin{cases} 0.55, & \text{if } y = 8 \\ 0, & \text{if } y \ne 8, \end{cases}$
- $\mu_{Q_3^*}(y) = J_3(\mu_{A4}(4.5), \mu_{\{6\}}(y)) = \begin{cases} \mu_{A4}(4.5) = 0.5, & \text{if } y = 6 \\ 0, & \text{if } y \ne 6, \end{cases}$

And finally, the output will be the $\mu_{Q^*}(y) = \max(\mu_{Q_1^*}, \mu_{Q_2^*}, \mu_{Q_3^*})$

So,

$$\mu_{Q^*}(y) = \begin{cases} 0.45, & \text{if } y = 2 \\ 0.5, & \text{if } y = 6 \\ 0.55, & \text{if } y = 8 \\ 0, & \text{if } y \in [0, 10] \setminus \{2, 6, 8\}. \end{cases}$$

Represented graphically in figure 8

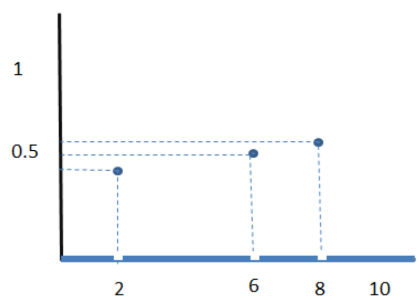

Figure 8: Output

4.4. More contents in High Schools

Taking into account the application to practical and theoretical problems, it is useful to show the existence of fuzzy number, that are the representation through fuzzy sets of the predicate "*being around the number n*". Also, they are possible to be operated thanks to Zadeh's Extension Principle [19] and it could be shown how to sum, or multiply fuzzy sets by a natural number.

In High schools students also has to deal with matrices. Matrices could represent relation between the elements in the row and in the columns, if the number of these matrices belong to the unit interval, these matrices can represent fuzzy relations. Then, in these last stage of basic education, fuzzy relation could be introduced, naturally following present programs.

5. Conclusions

This paper is nothing else than a proposal to study Fuzzy Logic in Basic Education. Fuzzy Logic is a new field that, under my view has to be popularized, as it happened with other sciences like Statistics. The reasons of that popularization are that it is a useful area in several disciplines and since its emergence, it has been proven its importance and its utility, mostly, in engineering. Anyway, its capability of collecting imprecision could be interesting also in Humanities and Social Sciences.

So, further of hide Fuzzy Logic, I think it would be crucial to show the potential of it. And, as a previous stage, to show that Maths, through a logic structures, allow to model imprecision, which is very important since the main characteristics of human beings are variability and originality of people and without imprecision they will disappear. In that way, Fuzzy Logic approaches Mathematics and Reality.

Even more, along the paper I mark some notions on Fuzzy Logic that could be introduced along the basic educational process. Since, for that purpose, difficult mathematical tools are not necessary.

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