*Proof.* By the property (NP) for the implications $J$ and $K$ we have

$$I(1,y) = J(1,K(1,y)) = J(1,y) = y$$

for any $y \in [0,1]$. □

**Theorem 13.** *If $I$ is a fuzzy implication satisfying property* (EP)*, then $(I,I)$-implication fulfils the same property.*

*Proof.* Let $x,y,z \in [0,1]$. From the formula (4) and the property (EP) for the fuzzy implication $I$ we have

$$\begin{aligned} I_{J,K}(x, I_{J,K}(y,z)) &= I_{J,K}(x, I(y, I(y,z))) \\ &= I(x, I(x, I(y, I(y,z)))) \\ &= I(x, I(y, I(x, I(y,z)))) \\ &= I(y, I(x, I(y, I(x,z)))) \\ &= I(y, I(y, I(x, I(x,z)))) \\ = I_{J,K}(y, I(x, I(x,z))) &= I_{J,K}(y, I_{J,K}(x,z)), \end{aligned}$$

which means that our $(I,I)$-implication fulfills (EP). □

**Theorem 14** ([4], p. 75, cf. [9])**.** *If $J$, $K$ are fuzzy implications, and $K$ fulfills* (IP)*, then $(J,K)$-implications fulfills* (IP)*.*

*Proof.* Let $x \in [0,1]$. By formula (4) and by Corollary 3 we obtain

$$I_{J,K}(x,x) = J(x, K(x,x)) = J(x,1) = 1$$

which imply that the implication $I_{J,K}$ fulfills (IP). □

**Example 9.** Let us observe that the implications $I_{I_{GD},I_{GD}}$, $I_{I_{RS},I_{RS}}$ inherit property (OP) of their generators. However a $(J,K)$-implication usually do not fulfil property (OP) even if its generators $J$ and $K$ do. Let us consider $(I_{GG},I_{GG})$-implications. We have

$$\begin{aligned} I_{I_{GG},I_{GG}}(x,y) &= I_{GG}(x, I_{GG}(x,y)) \\ &= \begin{cases} 1, & \text{if } x \le I_{GG}(x,y) \\ \frac{I_{GG}(x,y)}{x}, & \text{if } x > I_{GG}(x,y) \end{cases} \\ &= \begin{cases} 1, & \text{if } y \geqslant x^2 \\ \frac{y}{x^2}, & \text{if } y < x^2 \end{cases} \end{aligned}$$

for any $x,y \in [0,1]$. Let $x = 0.5$, $y = 0.3$. Then $I_{I_{GG},I_{GG}}(x,y) = I_{I_{GG},I_{GG}}(0.5, 0.3) = 1$, but $y < x$. Hence, $(I_{GG},I_{GG})$-implication does not fulfil (OP), despite the fact that $I_{GG}$ fulfils this condition.

## 5. Conclusion

In this contribution two ways of generating of fuzzy implication from other fuzzy connectives are considered. The first method, according to the formula (3), involves three kinds of weak fuzzy connectives: fuzzy conjunction, disjunction and negation. The results show some dependencies between properties of generators and properties of generated operation. There are still a lot of possibilities to complete these findings. The second method (formula (4)) uses two fuzzy implications as generators. Preservation of some properties are considered. There are other properties of fuzzy implications which can be examined, e.g. the law of left or right contraposition with respect to some fuzzy negation.

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