for some $\mu_k \in [x_{k-5}, x_{k+5}]$.

From the fact that function $(f_{F,n}^{(3)})''$ on interval $[x_{k-1}, x_k]$ is a linear we obtain for $x \in [x_{k-1}, x_k]$

$$(f_{F,n}^{(3)})''(x) =$$
$$= (f_{F,n}^{(3)})''(x_{k-1})\frac{x_k - x}{h} + (f_{F,n}^{(3)})''(x_k)\frac{x - x_{k-1}}{h} =$$
$$= f''(\mu_{k-1})\frac{x_k - x}{h} + f''(\mu_k)\frac{x - x_{k-1}}{h}.$$

Now we are ready to compare values $(f_{F,n}^{(3)})''(x)$ and $f''(x)$ at arbitrary point $x \in [x_{k-1}, x_k]$:

$$f''(x) - (f_{F,n}^{(3)})''(x) =$$
$$= f''(x) - (f''(\mu_{k-1})\frac{x_k - x}{h} + f''(\mu_k)\frac{x - x_{k-1}}{h}) =$$
$$= (f''(x) - f''(\mu_{k-1}))\frac{x_k - x}{h} +$$
$$+(f''(x) - f''(\mu_k))\frac{x - x_{k-1}}{h}.$$

If $x \in [x_{k-1}, x_k]$ and $\mu_{k-1} \in [x_{k-5}, x_{k+3}]$, then

$$|f''(x) - f''(\mu_{k-1})| \leq \omega(6h, f'').$$

Conversely, if $x \in [x_{k-1}, x_k]$, $\mu_k \in [x_{k-4}, x_{k+4}]$, then

$$|f''(x) - f''(\mu_k)| \leq \omega(6h, f'').$$

Now we have

$$|f''(x) - (f_{F,n}^{(3)})''(x)| \leq \omega(6h, f'')\frac{x_k - x}{h} +$$
$$+\omega(6h, f'')\frac{x - x_{k-1}}{h} = \omega(6h, f'').$$

6. Conclusions

In this paper a new type of fuzzy partitions based on polynomial splines are considered. The main attention is paid to approximation properties of $F^{(m)}$–transforms defined with respect to spline based generalized fuzzy m-partition. We prove the error bounds for approximation of an original function and its derivatives in the context of the classical approximation theory.

The obtained results allow us to propose the hypothesis that for a given function f from $C^{m-1}([a_m, b_m])$, $m \geq 1$, its inverse $F^{(m)}$–transform $f_{F,n}^{(m)}$, $n \geq 2$, with respect to spline based uniform generalized fuzzy m-partition

$$S_k^{(m)},\ k = -m+2, -m+3, ..., n+m-1,$$

on $[a, b]$, where

$$a_m = a - (2m-1)h,\ b_m = b + (2m+1)h,$$
$$h = (b-a)/(n-1),$$

provides the following result for derivative approximation:

$$|f^{(m-1)}(x) - (f_{F,n}^{(3)})^{(m-1)}(x)| \leq \omega(2mh, f^{(m-1)})$$

for all $x \in [a, b]$, where $\omega(2mh, f^{(m-1)})$ is the modulus of continuity of $f^{(m-1)}$ on $[a_m, b_m]$ according to $2mh$.

At the present moment this error bound is proved for $m = 1, 2, 3$. Our future work will be devoted to the proving of the general case.

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