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Approximation of Homogeneous Linear Algebraic Transition Systems

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***Abstract*—In terms of approximation of labeled transition systems, it is inefficient to simplify system actions. This paper proposed approximation of homogeneous linear algebraic transition systems. In order to extend the system actions, homogeneous linear algebraic programs are applied to describe system actions. Then it proposed the theory and algorithm of approximation of homogeneous linear algebraic transition systems. Under certain conditions, approximation of homogeneous linear algebraic transition systems can optimize homogeneous linear algebraic transition systems efficiently. Finally, above view is verified by a simple example.**

Keywords-approximate; labeled transition system; homogeneous linear algebraic transition system

I. INTRODUCTION

A transition system is a traditional description system which describes the transition relation between states. A transition system was initially used for modeling a real-time system. Subsequently, it is massively used to apply in the data intensive system and integrated circuit modeling. In the 1976, a transition system concept is initially proposed by Keller and he used the system for modeling analysis of concurrent programs [1]. In the 1984, Nicola and Hennessy introduced the transition system into a nondeterministic modeling process and analyze the probability of transition system [2,3]. Cleaveland proposed the detection method which is established in probability detection process [4]. In the 2000, Rusu proposed the I/O automata theory [5].

A transition system is the most basic computer system model. It is the powerful tool for program description and system verification. A transition system can also serve as language semantics model of many formal languages, such as communication system calculus, communication order processes, time order description languages. The formal verification of the transition system is a research hotspot in consistency checking.

This paper is organized as follows. In section 2, the homogeneous linear algebraic transition system is proposed. In section 3, the approximation of homogeneous linear algebraic transition system is established. Through approximate method in matrix eigenvalues and matrix norms, we decide whether the trace is approximate or not. In section 4, above view is verified by a simple example. In section 5, conclusion is given.

II. HOMOGENEOUS LINEAR ALGEBRAIC TANSITION SYSTEMS

Definition 1 (Homogeneous Linear Algebraic program) Let R be the set of real numbers, $x_i (i=1,2,\cdots n) \in \mathrm{R}$ and $x_i' (i=1,2,\cdots n) \in \mathrm{R}$ be the variables, $a_{ij} (i=1,2,\cdots,n; j=1,2,\cdots,n) \in \mathrm{R}$ be the coefficient, a homogeneous linear algebraic program is an algebraic program likes $X' = AX$, where $X = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ and $X' = (x_1', x_2', \cdots, x_n')^{\mathrm{T}}$ are the pre- and post- state values of the homogeneous linear algebraic program transition, $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ is the invertible matrix. Let $F(X) = AX$ and $F = (f_1, f_2, \cdots, f_n)^{\mathrm{T}}$, then $f_i(X) = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n (i=1,2,\cdots,n)$.

Definition 2 (Labeled Transition System) A labeled transition system is a tuple $< s_0, S, L, \rho >$, where

1) s_0 is the initial state.
2) S is a finite set of states.
3) L is finite set of labels.
4) $\rho \subseteq S \times L \times S$ is a set of transitions. Each transition $r \in \rho$ is a tuple $< s_1, l, s_2 >$, where s_1 and s_2 respectively represent the pre- and post- states of the transition.

Definition 3 (Finite Execute Sequence) In the labeled transition system, a finite execute sequence is $\eta = s_0 l_1 s_1 \cdots l_n s_n$ and it satisfies $s_i \xrightarrow{l_{i+1}} s_{i+1}$, where $0 \le i \le n-1$.

Definition 4 (Infinite Execute Sequence) In the labeled transition system, a infinite execute sequence is $\eta = s_0 l_1 s_1 \cdots$ and it satisfies $s_i \xrightarrow{l_{i+1}} s_{i+1}$, where $0 \le i$.

Definition 5 (Finite Trace) In the labeled transition system, a finite trace is a finite action sequence $l_1 l_2 \cdots l_n$ and it satisfies a finite execute sequence $\eta = s_0 l_1 s_1 \cdots l_n s_n$.

Dedinition 6 (Infinite trace) In the labeled transition system, a infinite trace is a infinite action sequence $l_1 l_2 \cdots$ and it satisfies a infinite execute sequence $\eta = s_0 l_1 s_1 \cdots$.

Definition 7(Homogeneous Linear Algebraic Transition System) A homogeneous linear algebraic transition system is a tuple $< s_0, S, V, X_0, XS, L, \rho >$,where

1) s_0 is a initial state.
2) S is a finite set of states.
3) V is a set of variables.
4) X_0 is the initial state value.
5) XS is a finite set of state value.
6) L is a set of homogeneous linear algebraic programs. A homogeneous linear algebraic program $l \in L$ describes an action, $l: X' = AX = F(X)$, where X and X' are the pre- and post- state values of the transition.
7) $\rho \subseteq S \times L \times S$ is a set of state transition. $r \in \rho$ is a tuple $< s_1, l, s_2 >$,where s_1 and s_2 are the pre- and post- states of the transition.

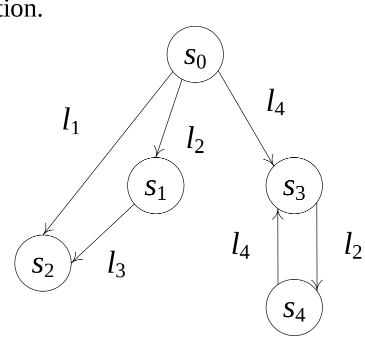

Figure 1. Example of a homogeneous linear algebraic transition system

In the Fig. 1, l_1, l_2, l_3, l_4 are homogeneous linear algebraic transition programs, s_0, s_1, s_2, s_3, s_4 are the system states.

III. Approximation Of Homogeneous Linear Algebraic Transition Systems

In the field of computer science and control science, matrixes are the results by measuring and computing. They often have some measuring errors and rounding errors. Let $A + \delta A$ be the real matrix, A be the approximate matrix. We will study the relation between real matrix $A + \delta A$ and approximate matrix A. In this section, we research the approximation of homogeneous linear algebraic transition systems by an initial state s_0 of homogeneous linear algebraic transition systems.

If a trace of a homogeneous linear algebraic transition system is successive composed by k same homogeneous linear algebraic programs, the real homogeneous linear algebraic program is $X' = (A + \delta A)X$, the approximate homogeneous linear algebraic program is $X' = AX$, then the total real homogeneous linear algebraic program is $X' = (A + \delta A)^k X$, the total approximate homogeneous linear algebraic program is $X' = A^k X$. The Jordan standard form of real matrix $A + \delta A$ is $P(A + \delta A)P^{-1} = \begin{pmatrix} A_1 + \delta A_1 & & \\ & \ddots & \\ & & A_t + \delta A_t \end{pmatrix} = A' + \delta A'$, the Jordan standard form of approximate matrix A is $PAP^{-1} = \begin{pmatrix} A_1 & & \\ & \ddots & \\ & & A_t \end{pmatrix} = A'$, the eigenvalue of matrix $(A + \delta A)^k$ is $(\lambda_i + \delta\lambda_i)^k$, the eigenvalue of matrix A^k is $\lambda_i^{\,k}$.

$$\frac{\left|(\lambda_i + \delta\lambda_i)^k - \lambda_i^{\,k}\right|}{\left|\lambda_i^{\,k}\right|} \approx \frac{\left|k\lambda_i^{\,k-1}\delta\lambda_i + \frac{k(k-1)\lambda_i^{\,k-2}}{2}\delta\lambda_i^{\,2}\right|}{\left|\lambda_i^{\,k}\right|} =$$

$\left|k\frac{\delta\lambda_i}{\lambda_i} + \frac{k(k-1)}{2}\left(\frac{\delta\lambda_i}{\lambda_i}\right)^2\right| = W_i$. If there exists a positive number ε, $\forall i, 1 \le i \le t$, it holds $W_i < \varepsilon$, then matrix $(A_i + \delta A_i)^k$ and matrix $A_i^{\,k}$ are approximate about ε.

Theorem 1 If program $X' = (A + \delta A)^k X$ and program $X' = A^k X$ are approximate, then two traces which are respectively successive composed by k same linear algebraic programs are approximate.

If a trace of a homogeneous linear algebraic transition system is successive composed by k different homogeneous linear algebraic programs, the real homogeneous linear algebraic program is $X' = (A_j + \delta A_j)X$, the approximate homogeneous linear algebraic program is $X' = A_j X$, then the total real homogeneous linear algebraic program is $X' = (A_k + \delta A_k)(A_{k-1} + \delta A_{k-1}) \cdots (A_1 + \delta A_1)X$, the total approximate homogeneous linear algebraic program is $X' = A_k A_{k-1} \cdots A_1 X$. The Jordan standard form of real matrix $(A_k + \delta A_k)(A_{k-1} + \delta A_{k-1}) \cdots (A_1 + \delta A_1)$ is $P(A_k + \delta A_k)(A_{k-1} + \delta A_{k-1}) \cdots (A_1 + \delta A_1)P^{-1} = A' + \delta A'$, the Jordan standard form of approximate matrix $A_k A_{k-1} \cdots A_1$ is $PA_k A_{k-1} \cdots A_1 P^{-1} = A'$. The i-th eigenvalue of matrix $(A_k + \delta A_k)(A_{k-1} + \delta A_{k-1}) \cdots (A_1 + \delta A_1)$ is $(\lambda_{k,i} + \delta\lambda_{k,i})(\lambda_{k-1,i} + \delta\lambda_{k-1,i}) \cdots (\lambda_{1,i} + \delta\lambda_{1,i})$, the i-th eigenvalue of matrix $A_k A_{k-1} \cdots A_1$ is $\lambda_{k,i}\lambda_{k-1,i} \cdots \lambda_{1,i}$.

$$\left|\frac{(\lambda_{k,i} + \delta\lambda_{k,i})(\lambda_{k-1,i} + \delta\lambda_{k-1,i}) \cdots (\lambda_{1,i} + \delta\lambda_{1,i})}{\lambda_{k,i}\lambda_{k-1,i} \cdots \lambda_{1,i}} - 1\right| \approx$$

$\left|\frac{\delta\lambda_{k,i}}{\lambda_{k,i}} + \frac{\delta\lambda_{k-1,i}}{\lambda_{k-1,i}} + \cdots + \frac{\delta\lambda_{1,i}}{\lambda_{1,i}}\right| = W_i$. If there exists a positive number ε, $\forall i, 1 \le i \le n$, it holds $W_i < \varepsilon$, then matrix $(A_k + \delta A_k)(A_{k-1} + \delta A_{k-1}) \cdots (A_1 + \delta A_1)$ and matrix $A_k A_{k-1} \cdots A_1$ are approximate about ε.

Theorem 2 Let $k \ge 1, k \ge j \ge 1$, all linear algebraic programs $X' = (A_j + \delta A_j)(A_{j-1} + \delta A_{j-1}) \cdots (A_1 + \delta A_1)X$ and

$X' = A_j A_{j-1} \cdots A_1 X$ are approximate if and only if two traces which are respectively in turn composed by k arbitrarily linear algebraic programs $X' = (A_j + \delta A_j)X$ and $X' = A_j X$ are approximate.

Theorem 3 If all traces of two homogeneous linear algebraic transition systems are approximate, then two homogeneous linear algebraic transition systems are approximate.

Approximation of homogeneous linear algebraic transition systems can reduce the coefficient bits of homogeneous linear algebraic programs.

IV. EXPERIMENTS

Let $X_0 = (100,10,10)^{\mathrm{T}}$ be the initial state value and $\varepsilon = 0.04$ be the error allowed value. The real homogeneous linear algebraic transition system is shown as Fig. 2.

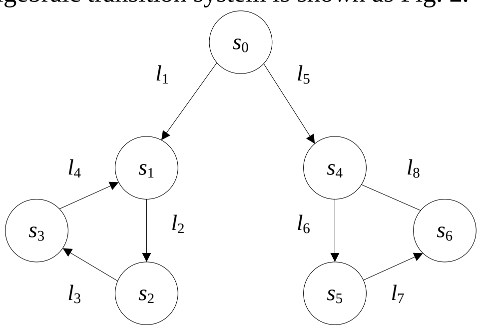

Figure 2. The real homogeneous linear algebraic transition system

l_1 is $X' = \begin{pmatrix} 1.001 & 0 & 0 \\ 0 & 1.001 & 0 \\ 0 & 0 & 1.002 \end{pmatrix} X$, l_2 is $X' = \begin{pmatrix} 2.01 & 0 & 0 \\ 0 & 2.02 & 0 \\ 0 & 0 & 2.01 \end{pmatrix} X$, l_3 is $X' = \begin{pmatrix} 4.01 & 0 & 0 \\ 0 & 2.02 & 0 \\ 0 & 0 & 2.01 \end{pmatrix} X$, l_4 is $X' = \begin{pmatrix} 1/8.0601 & 0 & 0 \\ 0 & 1/4.0804 & 0 \\ 0 & 0 & 1/4.0401 \end{pmatrix} X$, l_5 is $X' = \begin{pmatrix} 0.999 & 0 & 0 \\ 0 & 0.999 & 0 \\ 0 & 0 & 0.998 \end{pmatrix} X$, l_6 is $X' = \begin{pmatrix} 1.99 & 0 & 0 \\ 0 & 1.98 & 0 \\ 0 & 0 & 1.99 \end{pmatrix} X$, l_7 is $X' = \begin{pmatrix} 3.99 & 0 & 0 \\ 0 & 1.98 & 0 \\ 0 & 0 & 1.99 \end{pmatrix} X$, l_8 is $X' = \begin{pmatrix} 1/7.9401 & 0 & 0 \\ 0 & 1/3.9204 & 0 \\ 0 & 0 & 1/3.9601 \end{pmatrix} X$.

The approximate homogeneous linear algebraic transition system is shown as Fig. 3.

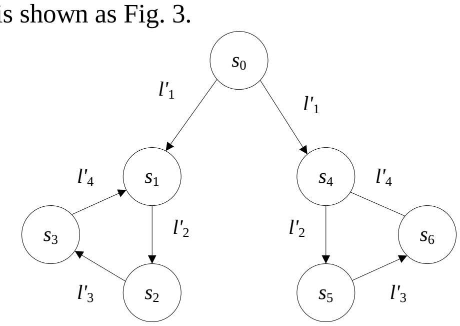

Figure 3. The approximate homogeneous linear algebraic transition system

l_1' is $X' = X$, l_2' is $X' = 2X$, l_3' is $X' = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} X$, l_4' is $X' = \begin{pmatrix} 0.125 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{pmatrix} X$.

$W_{\max} < \varepsilon$, all traces of two homogeneous linear algebraic transition systems are approximate, two homogeneous linear algebraic transition systems are approximate.

In the real homogeneous linear algebraic transition system, there have eight real homogeneous linear algebraic programs. In the approximate homogeneous linear algebraic transition system, there have only four approximate homogeneous linear algebraic programs. Approximation of homogeneous linear algebraic transition systems can optimize homogeneous linear algebraic programs.

V. CONCLUSION

In this paper, the approximation of homogeneous linear algebraic transition system is proposed. Under certain conditions, it can reduce coefficient bits of homogeneous linear algebraic programs. In the future work, we will study approximation of inhomogeneous linear algebraic transition systems.

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