$$\begin{pmatrix} (1-\cos\theta)\cdot c1^2+\cos\theta & -\sin\theta\cdot c3+(1-\cos\theta)\cdot c1\cdot c2 & \sin\theta\cdot c2+(1-\cos\theta)\cdot c1\cdot c3 \\ \sin\theta\cdot c3+(1-\cos\theta)\cdot c1\cdot c2 & (1-\cos\theta)\cdot c2^2+\cos\theta & -\sin\theta\cdot c1+(1-\cos\theta)\cdot c2+c3 \\ -\sin\theta\cdot c2+(1-\cos\theta)\cdot c1\cdot c3 & \sin\theta\cdot c1+(1-\cos\theta)\cdot c2+c3 & (1-\cos\theta)\cdot c3^2+\cos\theta \end{pmatrix} \quad (3)$$

7. Conclusions

The final formula for the free-rotation in the three dimensions is Simple and easy to use. The thinking of the derivation plays a significant role in the whole transformation process. Reference frame transformation has been applied to solve many complex problems such as z transform, w' transform.

8. References

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