Fuzzy Approaches Applied into Balanced Scorecard Customer Perspective

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Abstract

Mathematical approaches of the calculation of quality evaluation of the systems involving human factors do not reflect a significant feature of the processed data - their natural vagueness. The appropriate theoretical background for the formalization of data vagueness is the fuzzy set and fuzzy logic theory. In this paper, these approaches are presented and applied to one of the key performance indicators of customer perspective within the approach of Balanced Scorecard.

Keywords: Balanced Scorecard method, customer perspective, vagueness, fuzzy logic, fuzzy number, fuzzy arithmetic

1. Introduction

Using the fuzzy approach to formalize vague phenomena is a method which belongs to the Artificial Intelligence area [6]. Paper links vagueness with the Balanced Scorecard (BSC) and shows benefit for managerial decision-making [3] and paper shows methodology of how to formalize the vagueness of evaluation criteria; using fuzzy rule models [14]. Simulation is used to verify the efficiency.

This is needed as common tools are obsolete [2] and performance is in vague areas [1, 5, 8, 13].

2. Determination of customer satisfaction index

Customer satisfaction index (CSI) is usually discovered by statistical survey, while using scaled answers. As an example we can take a classification of customer satisfaction with certain products when we evaluate answers using the whole numbers lying in the interval <0,10> (0 means unsatisfied, 10 satisfied).

As supplementary information we ask for classification in groups which is multiple choice question distinguishing 1 – "I concentrate on new products and I am willing to pay for distinct products", 2 – "I buy regular products for regular price" and 3 – "I buy products on sale for low prices".

Let's assume the answers from the customers might not be definite, as each customer perceives the scale differently. The same level of satisfaction can lead different respondents to various answers. Also respondent is not precisely able to transform the whole width of approaches, feelings etc. into one precise number and sharp value of the scale.

We have to expect that two identically feeling respondents will not mark the same value of the offered scale, and at the same time, a respondent when asked repeatedly will choose an answer different from the one he marked before.

Conventional mathematical model for the determination of partial and global evaluation criteria we set a default CSI within a group is given by this relation

$$IS_h = \frac{1}{K_h}\sum_{j=1}^{K_h} A_{j,h} \tag{1}$$

where $A_{j,h}$ is the answer of the respondent j in the group h and K_h is the total number of respondents in the group h. The global customer satisfaction index is expressed as

$$IS = \frac{1}{K}\sum_{j=1}^{K} A_j \tag{2}$$

where K is the total number of respondents and A_i is a value of the answer of the respondent j from the total number respondents.

Since we assume that the respondents' answers are affected by vagueness, we will continue with the process of its determination how to express this vagueness and how to incorporate it in our consideration.

3. Determination of the uncertainty of crisp relations

Consider an ordinary analytic function with one output variable and multidimensional argument

$$\underline{y} = f(\underline{s}, \underline{x}) \tag{3}$$

where $\underline{s}$ and $\underline{x}$ are vectors of its parameters and arguments defined as an ordinary real numbers. The corresponding uncertainty function is the modified expression (3), where $\underline{\tilde{s}}$ and $\underline{\tilde{x}}$ are vectors of its fuzzy parameters and fuzzy arguments as fuzzy numbers, formalized and triangular fuzzy sets $\underline{\tilde{S}}$ and $\underline{\tilde{X}}$ [15]. The value of the dependent variable is then formalized as fuzzy number $\tilde{y}$ - fuzzy set Y. For example, a one-dimensional fuzzy function

$$\tilde{y} = f(\underline{\tilde{s}}, \tilde{x}) \tag{4}$$

To calculate the output value $\tilde{y}$ an algebra of fuzzy numbers must be used. The shape of fuzzy sets $\mu_{\tilde{B}}(y)$ as the result of any arithmetic operation f between fuzzy numbers $\mu(x_1),..., \mu(x_n)$ can be calculated according to the formula of Zadeh ́s extesional principle [11, 4]

$$\mu_{\tilde{B}}(y) = \sup_{y=f(x_1,...,x_n)} \min\left[\mu(x_1),., \mu(x_n)\right], if \quad \exists y = f(x_1,., x_n), 0 \quad else \tag{5}$$

3.1. Uncertainty of numbers

In a natural language, uncertainty of crisp numbers is expressed by verbal quantifiers, e.g. [*a* is *"about*" 5], [*b* is *"approximately"* 22]. The intensity of linguistic quantifiers, if the concept is known, is well-understood and effectively used by a person.

Uncertainty of a crisp number is usually formalized using the devices of fuzzy set mathematics – fuzzy sets [12]. The fuzzy set $\tilde{A}$ is defined as a representation which assigns each element x of the universe X to one number $\mu_{\tilde{A}}(x) \in \langle 0,1 \rangle$ according to the level of its pertinence to the set $\tilde{A}$ defined n universe X (Fig.1).

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x)) \;|\; x \in X\}, \quad \mu_{\tilde{A}}(x) \in \langle 0,1 \rangle, \quad \forall x \in X \tag{6}$$

At least usually piecewise approximated function $\mu_{\tilde{A}}(x) = f(x)$ is a membership function which unequivocally defines the fuzzy set $\tilde{A}$. In engineering practices Fig. 1 shows important triangular approximations.

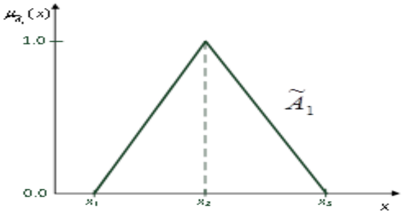

Fig. 1: Triangular Shape of Fuzzy Number.

Normal triangular fuzzy set $\tilde{A}_1$ formalizes then an uncertain number (fuzzy number) „approximately x_2". The rate of uncertainty of the number x_2 is given by the width of the carrier of the fuzzy set $\tilde{A}_1$ as a closed interval $\langle x_1, x_3 \rangle$ - see Fig.1. Parameters of such fuzzy sets form an ordered vector of break-points values $\tilde{A}_1\,[x_1, x_2, x_3]$. By using the vector, the fuzzy set (fuzzy number) $\tilde{A}_1$ can be formalized by a computer.

4. Approaches to the uncertainty modeling

To formulate and formalize the imprecise input/output dependences the fuzzy rule based model is used. Let us consider a fuzzy model of the Mamdani type, the structure of which is formed by a set of conditioned linguistic r = 1,2,…,R (*IF – THEN*) rules [12]. The form of the r-th rule of the Mamdani fuzzy model is

IF [x_1 is $A_r(x_1)$ and x_2 is $A_r(x_2)$ and x_n is $A_r(x_n)$] *THEN* [y is $B_r(y)$] (7)

Regarding the compositional formulas

$$B^0(y) = R \circ A(x_j^0) \tag{8}$$

where $B^0(y)$ is global output determined from model *R*, , $\circ$ is fuzzy compositional relation and $A(x_j^0)$ is a vector of *n*- actual sampled values of input variables, j = 1 ,…, n. To calculate the shape of output variable membership function the Mamdani inference is applied [12].

The output linguistic value $B^0(y)$ is defuzzified by the method of COF (Center of Gravity) into the form of an ordinary number y^{crisp}

$$y^{crisp} = \frac{\int_Y y.\mu_{B_y}\, dy}{\int_Y \mu_{B_y}\, dy} \tag{9}$$

The Mamdani fuzzy rule-based models are used to determine the size of respondent's degree of fuzzy-belonging to the group and to fuzzify its evaluation. The synthesis of appropriate linguistic models are performed taking into account the consequences of expert ́s hypotheses.

4.1. Fuzzification of the respondent's group belonging

Sharp belonging of the respondent *j* to the group *h* (respondent included into the group as *"exactly 1"*) is as following

$$k_{j,h}^{NAL} = 1 \tag{10}$$

Uncertain belonging (the respondent *j* included into the group as *"approximately 1"*)

$$\tilde{k}_{j,h}^{NAL} = \tilde{1} \text{ "}\textit{approximately } 1\text{"} \tag{11}$$

The uncertainty of the belonging of the respondent j ($j = 1,2,..., K_h$) to the group *h* (*h* = 1,2,3) is formalized by a triangular fuzzy number $\tilde{k}_{j,h}^{NAL}$, Fig. 2.

In case of zero uncertainty then (10) is applicable, in case of uncertainty, the belonging of a respondent to a group is given by a fuzzy number (11).

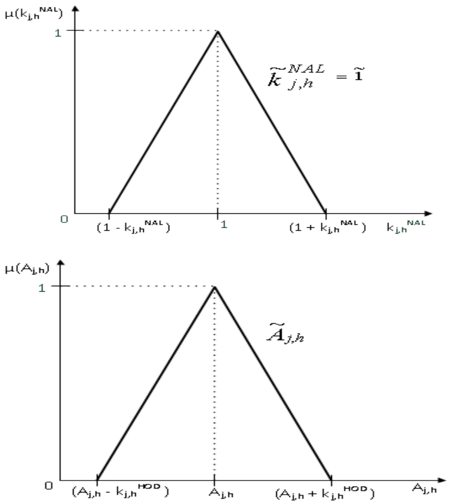

Fig. 2: Fuzzy Numbers of the Amount and Evaluation of Respondents.

According to Fig. 2, the fuzzy number $\tilde{k}_{j,h} \equiv \tilde{1}$ is its core "1" (Ker) and two partial fuzzy intervals ($\Delta_L \tilde{k}_{j,h}^{NAL}, \Delta_P \tilde{k}_{j,h}^{NAL}$) defined as follows

$$\tilde{k}_{j,h}\left\{[(1-k_{j,h}^{NAL})],1,[(1+k_{j,h}^{NAL})]\right\} = \tilde{k}_{j,h}\left\{[(\Delta_L \tilde{k}_{j,h}^{NAL})],1,[(\Delta_P \tilde{k}_{j,h}^{NAL})]\right\} \quad (12)$$

The rate of uncertainty $k_{j,h}^{NAL}$, Fig. 2, is given by a fuzzy model for each respondent separately considering the hypotheses and according to his evaluation of satisfaction $A_{j,h}$. The value of $k_{j,h}^{NAL}$ is taken as a consideration of the contributions from all the involved hypotheses.

The respondent is then included in the amount of respondents in the group h as a sum of the fuzzy units

$$\tilde{k}_h^{NAL} = \sum_{j=1}^{K_h} \tilde{1}_{j,h} \quad (13)$$

where K_h is a sharp number of respondents in the group h, h = 1, 2, 3.

4.2. Fuzzification of the respondent's evaluation

The fuzzified value of the evaluation of satisfaction of the respondent j in the group h $A_{j,h}$ is expressed by a fuzzy number $\tilde{A}_{j,h}$ [12] according to Fig. 2.

The rate of the evaluation uncertainty $k_{j,h}^{HOD}$ of the respondent j from the group h is calculated using the fuzzy model considering the contributions of uncertainty rates of the hypotheses considered. The triangular fuzzy number $\tilde{A}_{j,h}$ is written according to Fig. 2

$$\tilde{A}_{j,h} = \left\{[(A_{j,h} - k_{j,h}^{HOD})], A_{j,h}, [(A_{j,h} + k_{j,h}^{HOD})]\right\} \quad (14)$$

The index of customer satisfaction of the group h is a triangular fuzzy number $\tilde{IS}_h$, formalized by a fuzzy set with a triangular membership function

$$\tilde{IS}_h\left\{[IS_h - k_{j,h}^{HOD}], IS_h, [IS_h + k_{j,h}^{HOD}]\right\} = \tilde{IS}_h\left\{[\Delta_L \tilde{IS}_h], IS_h, [\Delta_P \tilde{IS}_h]\right\} \quad (15)$$

The rate of index uncertainty IS_h is given by the sum of the left and right indeterminate intervals

$$\Delta \tilde{IS}_h = \Delta_L \tilde{IS}_h + \Delta_P \tilde{IS}_h \quad (16)$$

The global index of customer satisfaction is also a triangular fuzzy number $\tilde{IS}$, formalized by a fuzzy set with a triangular membership function

$$\tilde{IS}\{[(IS-k_j)], IS, [(IS+k_j)]\} =$$
$$IS\{[(\Delta_L \tilde{IS})], IS, [(\Delta_P \tilde{IS})]\} \quad (17)$$

The rate of index uncertainty $\tilde{IS}$ is given by the sum of the left and right indeterminate intervals

$$\Delta \tilde{IS} = \Delta_L \tilde{IS} + \Delta_P \tilde{IS} \quad (18)$$

To evaluate the rate of index uncertainty we have possibility to compare results of two statistical surveys A and B.

4.3. Fuzzification of evaluation indices

The fuzzified index of customer satisfaction within groups (1) is now given by the relationship of the fuzzy multiplication of the sum of fuzzy numbers

$$\tilde{IS}_h = \frac{1}{\tilde{K}_h}\sum_{j=1}^{K_h} \tilde{A}_{j,h}, \quad h = 1,2,3 \quad (19)$$

The global fuzzified index of customer satisfaction (2) is now given by

$$\tilde{IS} = \frac{1}{\tilde{K}}\sum_{j=1}^{K} \tilde{A}_j \quad (20)$$

where $\tilde{A}_j$ is a fuzzified evaluation of the respondent j and K is the total number of respondents.

The values k_j^{HOD}, k_h^{HOD}, and k are defined by fuzzy models. The calculation of the relationships (19) and (20) is carried out with the help of fuzzy arithmetic [11, 4].

4.4. Hypothesis for the indices uncertainty estimation

To quantify the uncertainty of the correctness of respondents' answers, we expressed 4 expert fuzzy-logical rules, formalizing the existing facts (evidence) and their impacts (hypotheses) in the form $E \rightarrow H$. The relations for estimating the uncertainty of customer satisfaction indices as fuzzy numbers are also expressed.

1. Customers overestimate themselves (include themselves in a higher group) → the number of customers in the group h = 1 and h = 2 is excessive. The uncertainty of the respondent's belonging to groups h = 1 and h =2 is the higher, the higher or lower the level of its evaluation is. The uncertainty of the respondent's belonging to group h = 3 is low.

2. Exhibitionists are included in the set (in the group h = 1, there are more positive exhibitionists, in the group h = 3, there are more negative exhibitionists) → the amount of customers in the groups 1 and 3 are excessive. The uncertainty of the respondent's belonging to the group h = 1 is the higher, the higher the level of his evaluation is. The uncertainty of the respondent's belonging to the group h = 3 is the higher, the lower the level of his evaluation is. The uncertainty of the respondent's belonging to the group h = 2 is low.

3. Chronic complainers are more likely to answer, not preferring any appreciation → Lower evaluation levels of customer satisfaction in all groups are excessive. The uncertainty of the evaluation at a low rate in all groups is increased.

4. An average client does not support researches too much, generally, it is the exhibitionists who speak (answers are rather overestimating or underestimating) → The number of favourable evaluations in the group 1 and unfavourable in the group 3 is excessive. The uncertainty of the correctness of the level of the respondent's evaluation to the group 1 is

the higher, the higher the level of his evaluation is. The uncertainty of the level correctness of the respondent to the group 3 is the higher, the lower the level of his evaluation is.

These hypotheses represent mental models formulated linguistically, which will be formalized by computer through the fuzzy rule models in the following chapter.

4.5. Fuzzy models for the estimation of the level indices uncertainty

To derive the fuzzification rate of the respondent's belonging to a group and the fuzzification of the evaluation of his satisfaction, the Mamdani fuzzy rule models were designed [12] which respect expert hypotheses.

The FA_BSC system contains 3 language fuzzy models for individual groups of respondents (h = 1, 2, 3) – see Tab.1.

The input linguistic variable THE RESPONDENT'S EVALUATION IN GROUPS *HODRh* has three linguistic values: Low (NIZ), Middle (STR) and High (VYS). They are formalized by three fuzzy sets – see Fig.3.

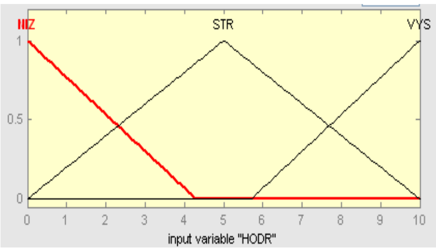

Fig. 3: Linguistic Values Membership Functions of Input Variable HODR.

The output linguistic variables THE RATE OF THE RESPONDENT'S BELONGING TO A GROUP *KNAL1* and THE RATE OF THE EVALUATION OF A RESPONDENT IN A GROUP *KHOD1* has four linguistic values: Low (MAL), Lowered (SN), Increased (ZVY), and High (VEL), formalized by four fuzzy sets (Fig.4).

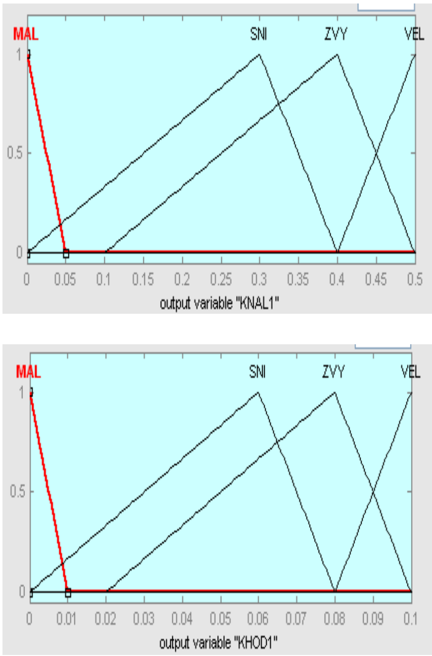

Fig. 4: Linguistic Values Membership Functions of Output Variables KNAL1, HOD1.

The membership function of the linguistic values of the output variables *KNAL2* and *KNAL3*, *KHOD2* and *KHOD3* are identical.

The rules of the fuzzy models (knowledge bases), deriving the output variables *KNALh*, respect the impacts of the hypotheses 1 and 2, the rules, deriving the output variables *KHODh*, respect the impacts of the hypotheses 3 and 4.

The FA_BSC system (Tab.1) includes the rules for three fuzzy models *FA_BSC_H1*, *FA_BSC_H2*, and *FA_BSC_H3*, each of which is assigned for the estimation of the fuzzification rate of the level of the respondent's belonging to the group *KNALh* and the fuzzification rate of his evaluation of satisfaction *KHOD$_j$h* for individual respondents j = 1, 2, ..., K_h in the groups h = 1, 2, 3.

FA_BSC_H1

R1 IF (*HODR1* is *NIZ*) THEN (*KNAL1* is *SNI* and *KHOD1* is *SNI*)

R2 IF (*HODR1* is *STR*) THEN (*KNAL1* is *MAL* and *KHOD1* is *MAL*)

R3 IF (*HODR1* is *VYS*) THEN (*KNAL1* is *VEL* and *KHOD1* is *SNI*)

FA_BSC_H2

R1 IF (*HODR2* is *NIZ*) THEN (*KNAL2* is *SNI* and *KHOD2* is *SNI*)

R2 IF (*HODR2* is *STR*) THEN (*KNAL2* is *MAL* and *KHOD2* is *MAL*)

R3 IF (*HODR2* is *VYS*) THEN (*KNAL2* is *SNI* and *KHOD2* is *MAL*)

FA_BSC_H3

R1 IF (*HODR3* is *NIZ*) THEN (*KNAL3* is *SNI* and *KHOD3* is *VEL*)

R2 IF (*HODR3* is *STR*) THEN (*KNAL3* is *MAL* and *KHOD3* is *MAL*)

R3 IF (*HODR3* is *VYS*) THEN (*KNAL3* is *MAL* and *KHOD3* is *MAL*)

Table 1: Structure of fuzzy models.

The creation of a language model is easy, the compilation of a mathematical model requires experts. Into the structure of the language model, the expert can easily involve any other rule which he himself followed in his mental model during the estimation of uncertainty. The fuzzy models FA_BSC_H1, FA_BSC_H2, and FA_BSC_H3 are implemented in the programming environment of the Fuzzy Toolbox package MATLAB [7].

Fig. 6 shows graphical analytical courses of the dependencies of output variables *KNALh* and *KHODh* for individual groups of respondents, h = 1, 2, 3. The graphs were derived from the fuzzy models Tab.1. Shapes in Fig. 6 correspond with source hypotheses.

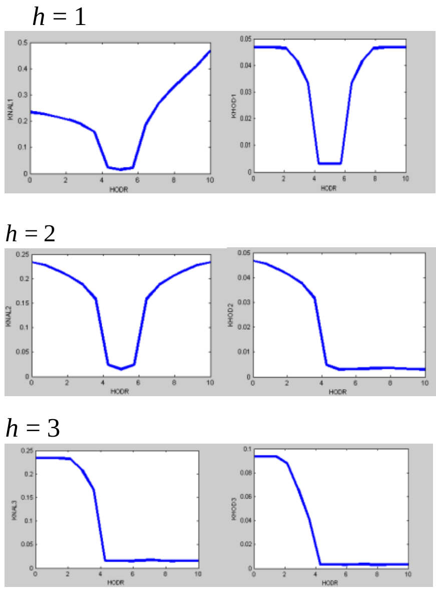

Fig. 6: Graphical Functional Dependencies of the Linguistic Variables.

5. Numerical calculation of fuzzified criteria based upon experimental data files

For the verification of fuzzy models functions, two data files with the evaluation of 30 respondents in 3 groups h = 1, 2, 3 were deliberately generated – Table 2. The *Data 1* file includes sub-files with higher levels of evaluation in the marginal areas of the scale (grey cells), leading, according to the expressed hypotheses, to the indices of satisfaction with a higher uncertainty. The *Data 2* file is formed by sub-files without a higher level of evaluation in the marginal areas of the scale, leading to the satisfaction indices with a lower uncertainty.

To calculate the BSC fuzzified criteria, the programming system for fuzzy arithmetic FA v1.00 has been used. The algorithms correspond to the Zadeh's extension principle and use the method of alpha-cuts [10].

-	DATA 1			DATA 2		
j	$A_{j,1}$	$A_{j,2}$	$A_{j,3}$	$A_{j,1}$	$A_{j,2}$	$A_{j,3}$
1	1	2	1	4	7	6
2	0	1	1	4	6	6
3	2	2	0	5	5	5
4	0	2	2	7	4	6
5	3	0	1	6	4	4
6	3	0	3	7	7	7
7	3	4	8	6	6	6
8	3	3	8	4	6	4
9	4	4	7	4	5	4
10	4	4	6	6	4	4
11	5	3	8	6	3	6
12	4	4	8	7	7	6
13	3	4	8	7	6	6
14	4	2	6	6	6	7
15	6	4	5	6	7	5
16	4	3	7	6	6	5
17	5	3	6	5	5	6
18	5	2	8	5	4	7
19	5	3	7	4	4	6
20	4	3	6	7	6	5
21	5	2	7	6	5	5
22	7	3	6	5	5	4
23	6	3	7	7	5	4
24	10	9	10	7	6	4
25	7	9	9	6	7	3
26	10	10	10	5	5	4
27	9	10	9	5	4	5
28	10	9	9	6	4	6
29	10	8	9	6	5	5
30	8	10	8	7	6	5

Table 2: Experimental data files.

On the following Fig.7 the graphical outputs of the calculation program FA are shown. The shapes of the membership functions of the fuzzy global evaluation IS (labeled as *ISx*) are stated, both for the data from the *DATA 1* file (the upper one), and for data from the *DATA 2* file (the lower one).

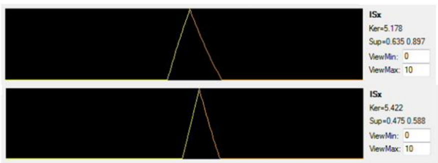

Fig. 7: Membership Function of the Fuzzy Numbers of Global Evaluation.

The legend is on the right side of the figures. The level of uncertainty of a particular fuzzy number is represented by the width of the carrier of their membership functions ΔIS . According to the characteristics of data files the global uncertainty of index *IS* calculated from the first file DATA1 (ΔS_{GLOB}= 1,532) is greater than this one calculated from the file DATA2 (ΔS_{GLOB}= 1,063).

6. Conclusions

This work presents the methodology of creating linguistic fuzzy models for uncertainty measures determination within the Balanced Scorecard framework, built on the basis of subjective expert hypotheses.

It shows formalization of hypotheses based on IF- THEN fuzzy models and respondents' answers in the form of fuzzy numbers. Numerical calculations show accuracy and efficiency of the proposed analytical algorithms, fuzzy algorithms and fuzzy models. Uncertainty measure informs the managers about their vagueness.

From the managerial perspective these models show how to process and numerically express the vagueness of Balanced Scorecard. Fuzzy type of uncertainty is therefore important attribute of the managerial information.

7. Acknowledgements

This work has been supported by Project GACR P403-12-1811: Unconventional Managerial Decision making Methods Development in Enterprise Economics and Public Economy.

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