

Optimal Policy for Deteriorating Items Ramp Type Demand with Shelf Life

KaiWayne Chuang

Department of Business Administration, Nanhua
University, Chiayi County, Taiwan.
e-mail: kwchuang@mail.nhu.edu.tw

ChienNan Lin

Department of Business Administration, Nanhua
University, Chiayi County, Taiwan.
e-mail: x4pili@yahoo.com.tw

ChunHsiung Lan

Department of Business Administration, Nanhua University, Chiayi County,
Taiwan.
e-mail: 074003@mail.hwc.edu.tw

Abstract—In this article, we have studied the inventory and pricing strategies for deteriorating items with ramp type demand and shelf life. In practice, there are many commodity such as food, medication exist the shelf life expiration date. Therefore the shelf life expiration date has become an important research topic in inventory management. The objective is to find the optimal inventory maximizing the net present value of total profit over the infinite horizon. We will discuss two cases. We assume that the inventory objective is to minimize the total cost per unit time of the system.

Keywords- ramp type demand; deteriorating items; backlogging; shelf life.

I. INTRODUCTION

The effect of deterioration is very important in inventory management. In order to avoid the loss due to damage or expiration, it is importance to develop a proper inventory strategy. Most of the literature assumes that a constant proportion of items will deteriorate per time-unit while they are in storage. Ghare and Schrader established a model for an exponentially decaying inventory [1]. Covert and Philip used a variable deterioration rate of two-parameter Weibull distribution to formulate the model with the assumption of a constant demand rate without shortages [2]. Shah and Jaiswal presented an order-level inventory model for deteriorating items with a constant rate of deterioration [3]. Aggarwal developed a note on an order-level model for systems with a constant rate of deterioration [4]. Other researchers, such as Skouri and Papachristos discussed optimal stopping and restarting production times for an EOQ (Economic Order Quantity) model with deteriorating items and time-dependent partial backlogging [5]. Jaggi, Aggarwal, and Goel studied optimal order policy for deteriorating items with inflation induced demand [6]. Teng, Ouyang, and Chen studied a comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items [7].

In reality, the demand for physical goods may be time-dependent, stock-dependent and price-dependent. Dave and Patel studied the inventory model for deteriorating items with time-varying demand [8]. Hill presented the inventory models for increasing demand followed by a constant demand [9]. Mandal and Pal extended the inventory model with ramp type demand to deterioration items, also allowing shortages [10]. Manna and Chaudhuri studied the EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages [11]. Other researchers, there are many literatures that propose and evaluate the algorithms [12], [13], [14], [15], [16], [17], [18].

This study proposes a single-supplier, single-retailer optimal inventory for deteriorating items with ramp type demand and shelf life. Shelf life is the recommendation of time that products can be stored, during which the defined quality of a specified proportion of the goods remains acceptable under expected conditions of distribution, storage and display. Most shelf life dates are used as guidelines based on normal and expected handling and exposure to temperature. In practice, there are many commodities such as food, medication, therefore; the existence of shelf life expiration date has become an important research topic in inventory management. It is important to control and maintain the inventories of deteriorating items for the modern corporation. Besides, expressions for an inventory system's total cost are derived for the two cases described below. Sensitivity analysis of the optimal solution with respect to major parameters is carried out. Finally, a numerical example is presented to demonstrate the developed model and the solution procedure. The results can help managers determine the optimal total cost.

II. NOTATIONS AND ASSUMPTIONS

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

A. Notations

C_1 setup cost per setup.

- C_2 unit holding cost per unit time.
- C_3 unit deteriorating cost per unit time.
- C_4 unit product expires lost cost.
- C_5 unit cost of lost sales.
- t_1 point of time when inventory level is maximum.
- t_2 point of time when all inventory is consumed.
- $t_1 + f$ point of time when the shelf life expiration date.
- t_3 the production restarting time. (i.e., Model 2).
- t_4 inventory cycle time (i.e., Model 2).
- $I(t)$ on-hand inventory at time t over $[0, t_4]$.
- I_{max} the maximum inventory level.
- OC ordering cost per cycle.
- HC unit total holding cost per cycle.
- DC unit deteriorating cost per cycle.
- EC cost of lost product.
- SC cost of lost sales per cycle.
- TC_1 total cost for a production cycle. (i.e., Model 1).
- TC_2 total cost for a production cycle. (i.e., Model 2).
- TVC_1 total average cost. (i.e., Model 1).
- TVC_2 total average cost f. (i.e., Model 2).

B. Assumptions

In addition, the following assumptions are used throughout this paper.

- 1) A single item is considered and infinite planning horizon.
- 2) There is a single-supplier and a single-retailer for a single product in this model.
- 3) There is no replacement or repair for over the intended shelf life expiration date.
- 4) The initial and final inventory levels are both zero.
- 5) Demand rate $R = D(t)$ is assumed to be a ramp type function of time, where is the function defined as follows:

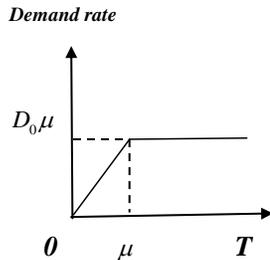


Figure 1. The ramp type demand rate

$$D(t) = \begin{cases} D_0 t & t \leq \mu \\ D_0 \mu & \mu \leq t \leq T \end{cases}$$

- 6) $P(t) = \beta D(t)$ is the production rate where β ($1 < \beta < 2$) is a constant.
- 7) Shortages are allowed to occur.

III. MATHEMATICAL MODEL AND SOLUTION

In this section, we will discuss two models: Model 1 is without shortage. The Model 2 is with shortage. Here the ramp type demand of an item is dependent on the relative size of μ .

A. Model 1. Model Without Shortage.

In this model 1, the production starts with zero stock level at time $t = 0$ and the production stops at time t_1 . Due to the combined effects of demand of items, the inventory level gradually diminishes during the period $[t_1; t_2]$, product expires at time $t_1 + f$, backlogging during the period $[t_1 + f; t_2]$ and ultimately falls to zero at time $t = t_2$.

The whole process is repeated and the behavior of the inventory system is depicted in "Fig.2". The inventory cycle here has the following four phases:

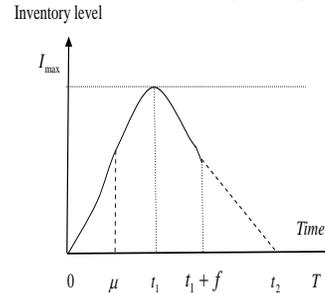


Figure 2. The inventory model without shortage.

Phase1. During the time interval $[0, \mu]$ the demand rate is $D_0 t$, the production rate is $\beta D_0 t$ and the deterioration rate is $\theta I_1(t)$ at time t . Therefore, the inventory level at time t , is governed by

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = (\beta - 1) D_0 t, \quad 0 \leq t \leq \mu. \quad (1)$$

Phase2. During the time interval $[\mu, t_1]$, from assumptions (5) and Figure 1 and "Fig.3", we know that the demand rate is D_0 the production rate is βD_0 , and the deterioration rate is $\theta I_2(t)$ at time t . Therefore, the inventory level at time t , is governed by

$$\frac{dI_2(t)}{dt} + \theta I_2 t = (\beta - 1)D_0 \mu, \quad \mu \leq t \leq t_1. \quad (2)$$

Phase3. In the time interval $[t_1, t_1 + f]$, the system is affected by the combined the demand and deterioration. Hence, the inventory level at time t , is governed by

$$\frac{dI_3(t)}{dt} + \theta I_3 t = -D_0 \mu, \quad t_1 \leq t \leq t_1 + f. \quad (3)$$

Phase4. In the time interval $[t_1 + f, t_2]$ the system is affected by the combined the demand and add the product expires and backlogging into the model. Hence, the inventory level at time t , is governed by

$$\frac{dI_4(t)}{dt} = -D_0 \mu, \quad t_1 + f \leq t \leq t_2. \quad (4)$$

The solution to “(1)” is

$$I_1(t) = \frac{e^{-t\theta}(\beta - 1)(1 - e^{t\theta} + e^{t\theta}t\theta)D_0}{\theta^2}, \quad 0 \leq t \leq \mu. \quad (5)$$

The solution to “(2)” is

$$I_2(t) = \frac{e^{-t\theta}(\beta - 1)(1 + e^{t\theta}\theta\mu - e^{t\theta})D_0}{\theta^2}, \quad \mu \leq t \leq t_1. \quad (6)$$

The solution to “(4)” is

$$I_4(t) = (-t + t_2)D_0 \mu, \quad t_1 + f \leq t \leq t_2. \quad (7)$$

The solution to “(3)” is

$$I_3(t) = -\frac{e^{-t\theta}(e^{t\theta} + e^{(f+t_1)\theta}(-1 + (f + t_1 - t_2)\theta))D_0 \mu}{\theta}, \quad t_1 \leq t \leq t_1 + f. \quad (8)$$

we have the maximum inventory level is given by $I_{\max} = I_3(t_1) = I_2(t_1)$.

$$\begin{aligned} & \frac{e^{-t_1\theta}(\beta - 1)(1 + e^{t_1\theta}\theta\mu - e^{t_1\theta})D_0}{\theta^2} \\ & = -\frac{e^{-t_1\theta}(e^{t_1\theta} + e^{(f+t_1)\theta}(-1 + (f + t_1 - t_2)\theta))D_0 \mu}{\theta}, \end{aligned} \quad (9)$$

From “(9)”, it is obvious that t_2 is a function of t_1 . As a result, the problem here has only one decision variable t_1 .

That is

$$t_1 = \frac{1}{\theta} [1 - e^{-t\theta} \beta - f\theta + t_2\theta + \text{ProductLog}[\frac{e^{-1+e^{-\theta} \beta - t_2\theta}(-1 + e^{t_2\theta})(-1 + \beta)}{\theta\mu}]]. \quad (10)$$

The total cost per cycle consists of the following three elements:

(a) The setup cost is $OC = C_1$.

(b) The inventory holding cost is given by

$$\begin{aligned} HC_1 &= C_2 [\int_0^{\mu} I_1(t)dt + \int_{\mu}^{t_1} I_2(t)dt + \int_{t_1}^{t_1+f} I_3(t)dt] \\ &= \frac{1}{2\theta^3} (e^{-t\theta} (2 + 2(e^{\theta\mu}(-1 + \beta) - 2\beta - 2e^{\theta(f+t_1)}\theta(-1 + (f + t_1 - t_2)\theta)\mu \\ &\quad - e^{t\theta}\theta\mu(2(\beta + t_2\theta - t_1\theta\beta) + (-1 + \beta)\theta\mu))C_2 D_0). \end{aligned} \quad (11)$$

(c) The cost of lost product over the intended shelf life per cycle is given by

$$EC_1 = C_3 I_3(t_1 + f) = -(f + t_1 - t_2)\mu C_3 D_0, \quad (12)$$

(d) The inventory deteriorating cost is given by

$$\begin{aligned} DC_1 &= C_4 [\beta \int_0^{\mu} D_0 t dt + \beta \int_{\mu}^{t_1} D_0 \mu dt - \int_0^{\mu} D_0 t dt - \int_{\mu}^{t_1} D_0 \mu dt - \int_{t_1}^{t_1+f} D_0 \mu dt] \\ &= -\frac{1}{2} \mu (2(f + t_1 - t_1\beta) + (-1 + \beta)\mu) C_4 D_0. \end{aligned} \quad (13)$$

(e) The cost of lost sales per cycle is given by

$$\begin{aligned} SC_1 &= C_5 \int_{t_1+f}^{t_2} I_4(-t + t_2)\mu D_0 dt \\ &= \frac{1}{2} (f + t_1 - t_2)^2 \mu C_5 D_0 \end{aligned} \quad (14)$$

Therefore, the total cost per unit time during time-span $[0, t_2]$ is given by

$$TC_1(t_1, t_2) = OC + HC_1 + EC_1 + DC_1 + SC_1 \quad (15)$$

$$TVC_1(t_1, t_2) = \frac{TC_1(t_1, t_2)}{t_2}. \quad (16)$$

Hence, the total relevant cost per unit time is a function of one variable t_2 because of “(16)”. The necessary condition for TVC_1 to be minimum is the optimal solution satisfies:

$$\frac{dTVC_1}{dt_2} \Big|_{(t_2)} = 0. \quad (17)$$

provided they satisfy the sufficient conditions

$$\frac{d^2TVC_1}{dt_2^2} \Big|_{(t_2)} > 0. \quad (18)$$

Due to the fact that t_1 is a function of t_2 , thus $TVC_1(t_1; t_2)$ in “(16)” can be reduced as a function of t_2 , we denoted it by $TVC_1(t_2)$, i.e., $TVC_1(t_2) = TVC_1(t_1; t_2)$. Hence, the problem faced by the vendor in Model 1 is

Minimize $TVC_1(t_2)$ Subject to: $0 \leq t_1 \leq t_2$. To minimize the total cost per unit time, taking the first derivative of $TVC_1(t_2)$ with respect to t_1 , and setting the result to be zero, we obtain $\frac{dTVC_1(t_2)}{dt_2} = 0$

Let t_1^* denoted the optimal value of t_2 , then t_2^* must satisfy “(18)”. Furthermore, we can see that the stationary point t_2 also satisfies the $\frac{d^2TVC_1}{dt_2^2} \Big|_{(t=t_2^*)} > 0$

Consequently, we can obtain the value of t_2^* from “(17)”, the numerical examples in section 4 below indicate so. Once the optimal solution t_2^* is obtained, the corresponding optimal value t_1^* can be determined from “(9)”.

B. Model 2. Model with shortage

In this model 2, the behaviors of inventory system is depicted in “Fig.3”.

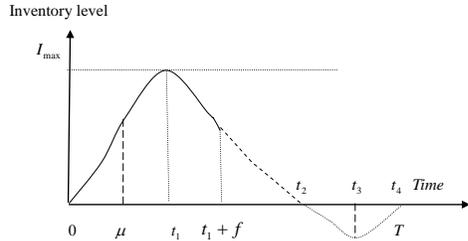


Figure 3. The inventory model with shortage.

The Phases 1, 2, 3 and 4 are same as those in Model 1.

Phase5. During the shortage interval $[t_2, t_3]$, the demand at time t is backlogged. Thus, the inventory level at time t is governed by the following differential equation:

$$\frac{dI_5(t)}{dt} = -D_0\mu, \quad t_2 \leq t \leq t_3. \quad (19)$$

The solution of “(19)” is

$$I_5(t) = (-t + t_2)D_0\mu, \quad t_2 \leq t \leq t_3. \quad (20)$$

Phase6. During the shortage interval $[t_3, t_4]$ the backorders level at time t , is governed by the following differential equation:

$$\frac{dI_6(t)}{dt} = (\beta - 1)D_0\mu, \quad t_3 \leq t \leq t_4. \quad (21)$$

The solution to “(21)” is

$$I_6(t) = (t - t_4)(\beta - 1)D_0\mu, \quad t_3 \leq t \leq t_4 \quad (22)$$

Given the condition $I_5(t_3) = I_6(t_3)$, we get

$$(-t_3 + t_2)D_0\mu = (t_3 - t_4)(\beta - 1)D_0\mu. \quad (23)$$

From “(23)”, it is obvious that t_2 is a function of t_3 . As a result, the problem here has only one decision variable t_3 .

$$t_2 = \frac{t_3 - t_4 + t_4\beta}{\beta}, \quad (24)$$

The total cost per cycle of the system consists of the following five elements:

- (a) The setup cost is $OC = C_1$.
- (b) The inventory holding cost is $HC_2 = HC_1$.
- (c) The inventory deteriorating cost is $DC_2 = DC_1$.
- (d) The cost of lost product over the intended shelf life is $EC_2 = EC_1$.
- (e) The cost of lost sales is $SC_{21} = SC_1$
- (f) The cost of lost sales is given by

$$\begin{aligned} SC_{22} &= C_5 \left[\int_{t_2}^{t_3} -I_4(t) dt + \int_{t_3}^{t_4} -I_5(t) dt \right] \\ &= \frac{1}{2} \left((t_2 - t_4)(t_2 - 2t_3 + t_4) + (t_3 - t_4)^2 \beta \right) C_5 \mu D_0 \end{aligned} \quad (25)$$

Consequently, the total cost per unit time during time-span $[0, t_4]$ is given by

$$TC_2(t_1, t_2, t_3, t_4) = OC + HC_2 + DC_2 + EC_2 + SC_{21} + SC_{22}. \quad (26)$$

where

$$TVC_2(t_1, t_2, t_3, t_4) = \frac{TC_2(t_1, t_2, t_3, t_4)}{t_4} \quad (27)$$

From “(9)” and “(23)”, we know that t_2 is a function of t_1 and t_4 is a function of t_2 and t_3 . Consequently, the decision variables in Model 2 can be reduced from four dimensions (t_1, t_2, t_3, t_4) to two dimensions (t_1, t_3) , i.e., the problem faced by the vendor in this model is

$$\text{Minimize: } TVC_2(t_1; t_3) \quad (28)$$

$$\text{Subject to: } 0 \leq t_1 \leq t_2, t_2 \leq t_3 \leq t_4.$$

Our objective is to find the optimal values of t_1 and t_3 such that $TVC_2(t_1; t_3)$ has minimum. That is, in order to find the optimal values of t_1 and t_3 , we have to solve the complex nonlinear equations $\frac{\partial TVC_1(t_1, t_3)}{\partial t_1} = 0$ and $\frac{\partial TVC_1(t_1, t_3)}{\partial t_3} = 0$.

We can obtain the optimal values t_1^* and t_3^* . Once we obtain the optimal value $(t_1^*; t_3^*)$, the optimal solution $(t_2^*; t_4^*)$ is obtained from “(9)” and “(23)”.

IV. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

A. We let $C_1 = \$1000$ per setup, $C_2 = \$30$ /unit,

$C_3 = \$50$ /unit, $C_4 = \$20$ /unit, $C_5 = \$10$ /unit, $\mu = 1$, $D_0 = 3$, $\beta = 1.3$, $\theta = 0.03$, $f = 0.25$, in appropriate units. By using the subroutine FindRoot in commercial software Mathematica 5.2.

B. *Example 1:* For the model 1 with shortage, we obtain the optimal solutions for t_1^* , t_2^* and TVC_1^* are given as $t_1^* = 8.69653$, $t_2^* = 10.8609$, and $TVC_1^* = 217.163$

C. *Example 2:* For the model 2 without shortage, we obtain the optimal solutions for t_1^* ; t_2^* ; t_3^* ; t_4^* and TVC_2^* are given as $t_1^* = 3.01378$, $t_2^* = 3.73534$, $t_3^* = 7.25921$, $t_4^* = 19.0054$ and $TVC_2^* = 105.716$

D. Sensitivity Analysis

For studying the sensitivity analysis of the parameters on the proposed models, we changed (increasing or decreasing) the parameters by 25% and 50% and took one parameter at a time, kept the remaining parameters at their original values.

a) Example 1:

- (1) The optimal total cost per unit time, TVC_1^* increases while $C_1, C_2, C_3, C_4, C_5, \mu, D_0$ and β increase. But this trend is reversed for parameters θ and f .
- (2) A larger value of C_1 indicates a larger setup cost per setup, while the optimal total average cost per unit time is getting larger.
- (3) A larger value of C_2 indicates a larger unit holding

cost per unit time; while the optimal total average cost per unit time is getting larger.

- (4) A larger value of C_3 indicates a unit product expires lost cost per unit time, while the optimal total average cost per unit time is getting larger.
- (5) A larger value of C_4 indicates a unit deteriorating cost per unit time, while the optimal total average cost per unit time is getting larger.
- (6) A larger value of C_5 indicates a unit product lost cost per unit time, while the optimal total average cost per unit time is getting larger.
- (7) A larger value of μ parameter, while the optimal total average cost per unit time is getting larger.
- (8) a larger of θ deterioration rate, while the optimal total average cost per unit time is getting smaller.
- (9) A larger value of f parameters, while the optimal total average cost per unit time is getting larger.
- (10) A larger value of D_0 parameters, while the optimal total average cost per unit time is getting larger.
- (11) It is seen that the percentage change in the optimal total average cost per unit time is highly sensitive in parameters C_1, C_2, μ and D_0 .

b) Example 2:

- (1) The optimal total cost per unit time TVC_2^* increases while $C_1, C_2, C_3, C_4, C_5, \mu$, and D_0 increase. But this trend is reversed for parameters θ and f .
- (2) A larger value of C_1 indicates a larger setup cost, while the total average cost per unit time is getting larger.
- (3) A larger value of C_2 indicates a larger unit holding cost per unit time, while the total average cost per unit time is getting larger.
- (4) A larger value of C_3 indicates a unit product expires lost cost per unit time, while the optimal total average cost per unit time is getting larger.
- (5) A larger value of C_4 indicates a unit deteriorating cost per unit time, while the optimal total average cost per unit time is getting larger.
- (6) A larger value of C_5 indicates a unit product lost cost per unit time, while the optimal total average cost per unit time is getting larger.
- (7) A larger value of μ parameters, while the total average cost per unit time is getting larger.
- (8) A larger value of θ parameters, while the optimal total average cost per unit time is getting smaller.
- (9) A larger value of f parameters, while the optimal total average cost per unit time is getting smaller.

- (10) A larger value of D_0 parameters, while the total average cost per unit time is getting larger.
- (11) It is seen that the percentage change in the optimal total average cost per unit time is highly sensitive in parameters C_1, C_5, μ and D_0 .

V. CONCLUDING REMARKS

This study proposes a single-supplier, single-retailer optimal inventory for deteriorating items with ramp type demand and shelf life. Shelf life is the recommendation of time that products can be stored, during which the defined quality of a specified proportion of the goods remains acceptable under expected conditions of distribution, storage and display. In practice, there are many commodities such as food, medication. Therefore the existence of shelf life expiration date has become an important research topic in inventory management. This paper presented a methodology and provided numerical results related to ramp type demand rate to find the optimal inventory policy. Two numerical examples are given to illustrate the solution procedure and sensitivity analysis has been shown.

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