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Mathematical Simulation of Heat Transfer in Relaxing Media

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Abstract

We find a numerically-analytical solution of a boundary problem for the third-order partial differential equation, which describes the mass and heat transfer in active media.

We consider the problem of heat transfer in active media. The solution of this problem reduces to solution of the partial differential equation [1]

$$R\,\frac{\partial^2 T}{\partial^2 t}+a\,\frac{\partial T}{\partial t}+va\,\frac{\partial}{\partial x}\left(T+\frac{R}{a}\,\frac{\partial T}{\partial t}\right)-c\,\frac{\partial^2}{\partial x^2}\left(T+\tau_2\,\frac{\partial T}{\partial t}\right)=\left(1+\frac{R}{a}\,\frac{\partial}{\partial t}\right)Q(T),\quad(1)$$

where R, a, c, v, τ_2 are constants, $Q(T)=T^m$ $(m\geq 1)$, $T(x,t)$ is temperature.

In accordance with (1) the boundary conditions are written in the form:

$$\begin{aligned}&T(0,t)=0, && T'_x(l,x)=0,\\ &T(x,0)=\varphi(x), && T'_t(x,0)=\psi(x),\end{aligned}\qquad(2)$$

where φ, ψ are known functions.

It is worth noticing that in the special case, where $m=1$, $v=0$, $R\neq 0$, the boundary-value problem (1), (2) can be solved as

$$T(x,t)=\frac{2}{l}\sum_{n=1}^{\infty}\overline{T}_n^{(i)}(t)\sin(\lambda_n x)\qquad(i=1,2,3),$$

where

$$\overline{T}_n^{(1)}(t)=(k_n^{(1)}-k_n^{(2)})^{-1}\left[\left(\beta_n-\alpha_n k_n^{(2)}\right)e^{k_n^{(1)}t}-\left(\beta_n-\alpha_n k_n^{(1)}\right)e^{k_n^{(2)}t}\right]\qquad(\alpha_n>0),$$

$$\overline{T}_n^{(2)}(t)=e^{-\frac{\mu_n}{2R}t}\left[\alpha_n\cos\omega_n t+\frac{1}{\omega_n}\left(\beta_n+\frac{\alpha_n\mu_n}{2R}\right)\sin(\omega_n t)\right]\qquad(\alpha_n<0),$$

$$\overline{T}_n^{(3)}(t)=\left[\alpha_n+\left(\beta_n+\frac{\alpha_n\mu_n}{2R}\right)t\right]e^{-\frac{\mu_n}{2R}t}\qquad(\alpha_n=0),$$

$$k^{(1,2)}=\frac{1}{2R}(-\mu_n\pm\sqrt{\alpha_n}),\quad \alpha_n=\mu_n^2+4R\kappa_n,\quad \omega_n=\frac{1}{2R}\sqrt{-\alpha_n},$$

$$\mu_n=a-\frac{R}{a}+c\tau_2\lambda_n^2,\quad \kappa_n=1+c\lambda_n^2,\quad \lambda_n=\frac{\pi}{2l}(2n-1),$$

$$\left\{ \begin{array}{c} \alpha_n \\ \beta_n \end{array} \right\} = \int\limits_0^l \left\{ \begin{array}{c} \varphi(x) \\ \psi(x) \end{array} \right\} \sin(\lambda_n x)\; dx.$$

In general case $m > 1$ ($V = 0,\;\; R \neq 0$) the boundary-value problem (1), (2) is reduced to a nonlinear integral equation. To this end, we apply the finite integral transform

$$\overline{T}_n(t) = \int\limits_0^l T(x,t)\sin(\lambda_n x)\; dx, \tag{3}$$

to the problem (1), (2). After the transform (3) is applied, we finally obtain the Volterra–Hammerstein integral equation

$$T(x,t) = \nu(x,t) + \int\limits_0^t \int\limits_0^l T^m(\xi,\tau) G(\xi, x;\; t-\tau)\; d\xi d\tau, \tag{4}$$

where $\nu(x,t),\; G(\xi,x;\; t-\tau)$ are known functions of their arguments.

In order to construct the solution of (4) we can use the projective method. We have

$$T_{j\mu} = F_{j\mu} + \sum_{i=1}^{N}\sum_{k=1}^{M} c_{ikj\mu} T_{ik}^m \qquad (j = \overline{1,N};\quad \mu = \overline{1,M}), \tag{5}$$

where $F_{j\mu},\; c_{ikj\mu}$ are known constants.

After having solved the system (5), we get the solution of the problem (1), (2).

A finite-difference scheme for solution of the boundary-value problem (1), (2) can be represented in the form

$$Ru_{\bar{t}t} + au_{\breve{t}} - c\widehat{u}_{\bar{x}x} - c\tau_2(u_{\bar{x}x})_{\breve{t}} = \widehat{f} + \frac{R}{a}\, f_{\bar{t}} \tag{6}$$

(the notation is the same as in [2]).

The results of calculations show that, for the class of problems considered here, the method of reducing (1), (2) to the integral equation enables us to find an effective solution. These results allow us to obtain qualitative summaries about the temperature behaviour. In particular, the blow-up regime [3] exists.

In the case $v \neq 0$ the problem under consideration may be reduced to solution of the finite-difference equation

$$Ru_{\bar{t}t} + au_{\breve{t}} + va\left(\widehat{u}_{\breve{x}} + \frac{R}{a}\, u_{\breve{x}\breve{t}}\right) - c\widehat{u}_{\bar{x}x} - c\tau_2(u_{\bar{x}x})_{\breve{t}} = \widehat{Q} + \frac{R}{a}\, Q_{\breve{t}}.$$

All the symbols here are the same as in [2]. The numerical examples are given.

References

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