A Kind of GIOWA Method

ZHU Zhiming[1, a],CHEN Tao[2, b]

[1]School of Business, Hohai university,Nanjing 211106,China
2 School of Engineering & Management ,Nanjing university ,Nanjing 210093,China

[a]email: zhimingzhu@126.com, [b]email:chenbuqing@gmail.com

Keywords: FAG; triangular fuzzy number; GIOWA operator; decision

Abstract. In order to make multi-attribute decision better, the paper proposes a group decision making method based on linguistic evaluation scales of FAG. By the combination of fuzzy analytic hierarchy processes and triangular fuzzy number, it can solve the problem of the matrix compatibility test and the complex calculation and improve GIOWA operator on the language decision-making model at the same time. Therefore, the calculation on result is not a fuzzy number, but a definite value which can sort multi-attribute decision-making programs precisely and obtain a relatively satisfactory result which will have more practical value. Finally, the paper carries out an example analysis and comparison.

Introduction

AHP[1](analytic hierarchy process) is an effective tool and method of multi-attribute decision-making, and forecasting analysis. It usually divides the attributes of decision-making scheme into several levels. The elements of each attribute will get judgment matrix through multiple comparisons under the same rule. According to the judgment matrix, AHP is based on the key issues to determine the weight of attribute elements. GIOWA operator first proposed by Xu Zeshui et al. [2]is an easy decision-making method.In the literature [3] AHP did not analyze attributes, and the attribute weights of program were man-given. In this paper, a fuzzy AHP can test the matrix compatibility easily and come in each attribute weights meanwhile GIOWA method can obtain specific actual data to deserve a reasonable order, so that to avoid the same sort options, and then take other ways to carry out complementary decision and fine adjustment.

Theoretical research

Fuzzy linguistic evaluation scales are the fuzzy linguistic evaluation made by decision maker under the specific attributes of the program, whose linguistic evaluation sets S=(excellent, perfect, very good, good, fair, poor, very poor, awful, extremely poor).,where E= excellent, PF= perfect , VG= very good, G= good, F= fair ,P= poor ,VP= very poor, A= awful ,EP= extremely poor .Its corresponding expression Triangular fuzzy numbers are as follows, among which,

E>PF>VG>G>F>P>VP>A>EP. E＝[0.8,0.9,1];PF＝[0.7,0.8,0.9]; VG＝[0.6,0.7,0.8];G＝[0.5,0.6,0.7]; F＝[0.4,0.5,0.6];P＝[0.3,0.4,0.5]; VP＝[0.2,0.3,0.4];A＝[0.1,0.2,0.3]; EP＝[0,0.1,0.2].

The fundamental characteristics of GIOWA operator[2] is that data $\langle v_i, u_i, a_i \rangle$ has nothing to do with w_i, which only relates to the position of i in the accumulation process while weighted accumulation towards a_i is not based on its own value but on the v_i $(i \in N)$ among $\langle v_i, u_i, a_i \rangle$ corresponding to a_i . u_i generally refers to the property of the problem and can be expressed by words or value. v_i generally refers to the importance or characteristics of u_i, such as weight, sequence, performance, etc. and also can be expressed by words or value. a_i generally refers to the property or the other representations and can be expressed only by value, such as real number, interval number and triangular fuzzy number, etc.

It needs to measure whether the judgment matrix structured is reasonable based on the requirements of AHP judgment matrix’ s compatibility. So the literature [4] supposes the compatibility requirements matrix $B = (b_{ij})_{n \times n}$ meet:

(1) $b_{ii}=1,\forall i\in(1,2,\cdots,n)$;(2) $b_{ij}\cdot b_{ji}=1,\forall i,j\in(1,2,\cdots,n)$; (3) $b_{ij}\cdot b_{jk}=b_{ik},\forall i,j,k\in(1,2,\cdots,n)$.

Let be $w_i=\sqrt[n]{\prod_{j=1}^{n}b_{ij}}$, then

$$\frac{w_i}{w_j}=\sqrt[n]{\prod_{k=1}^{n}b_{ik}}\bigg/\sqrt[n]{\prod_{k=1}^{n}b_{jk}}=\sqrt[n]{\frac{b_{i1}.b_{i2}...b_{in}}{b_{j1}.b_{j2}...b_{jn}}}$$
$$=\sqrt[n]{\left(\frac{b_{i1}}{b_{j1}}\right).\left(\frac{b_{i2}}{b_{j2}}\right)...\left(\frac{b_{in}}{b_{jn}}\right)}=\sqrt[n]{b_{ij}.b_{ij}...b_{ij}}=b_{ij} \quad (1)$$

when $\lambda_{\max}=n$, the eigenvector $X=(x_1,x_2,\ldots,x_n)^T$ corresponding to the largest eigen value in the judgment matrix is the weight vector $w=(w_1,w_2,\ldots,w_n)^T$. As the general judgment matrix B may not be compatible matrix, it may be incompatible with the degree of a certain range, that is: $C(B)=\frac{\lambda_{\max}-n}{n-1}\le 0.1$. If B accord with the above condition, it can use the eigenvector $X=(x_1,x_2,\ldots,x_n)^T$ corresponding to B ' s largest eigen value as the weight vector $w=(w_1,w_2,\ldots,w_n)^T$, otherwise it should adjust the judgment matrix B , and normalize the weight vector w .

In accordance with this method, the calculation is very complex if the matrix is more than five order matrix. Then it can use a simple alternative method [3].

Decision making steps are as follows:

(1) For a multi-attribute decision making problem, there are the scheme set X, attribute set U, group decision making set D. Group decision maker judges the attributes, structures the judgment matrix, solves the weight vector of the attribute set $w=(w_1,w_2,\ldots,w_n)^T$ through FAHP and normalizes it to get the new weight vector $w'=(w'_1,w'_2,\ldots,w'_n)^T$.

(2) Decision maker $D_i\in D$ proposes the scheme $x_i\in X$ under the attribute situation $U_i\in U$ and fuzzy linguistic evaluation r_{ij}^k , obtains the fuzzy linguistic evaluation matrix R_k .

(3) Use GIOWA operator to accumulate the linguistic evaluation information in the i line of the matrix R_k get the comprehensive attribute evaluation $r_i^{(k)}$ ($i=1,2,\cdots,n;k=1,2,\cdots,n$)of the decision scheme x_i by decision maker D_k . As a result, it can obtain the comprehensive attribute evaluation information according to each scheme fixed by decision maker D_k , so that to get specific figures instead of fuzzy numbers decision.

(4) Let w^* be different types of decision makers' weight, and then use the GIOWA operator to accumulate the comprehensive attribute evaluation $z_i^{(k)}(w)$, $(k=1,2,\cdots,n)$ according to the schemes x_i fixed by different types of decision makers D_i to get the comprehensive group attribute evaluation $z_i(w^*)$, $(i=1,2,\cdots,n)$ so that to get specific figures instead of fuzzy numbers decision.

(5) Compare, sort and prefer the comprehensive group attribute evaluation $z_i(w^*)$, $(i=1,2,\cdots,n)$

Numerical examples

A Space Research Institute prepares for a technological transformation of production equipment. There are four options which can be chosen x_i , $(i=1,2,3,4)$. The important indicators (attributes) decision making alternatives considered include: U_1 : the cleanliness of the environment; U_2 : the quality of the equipment running; U_3 :the implementation of technology; U_4 : the economy energy; U_5 : enterprise efficiency. There are three types of decision makers D_k , $(k=1,2,\cdots,n)$, (for example: technical staff, manager, production staff) and three evaluation matrixes (see Table 1.1 to 1.3) according to comprehensive analysis of each alternatives indicator .The problem is how to determine the best technical alternatives.

Step1: Three types of staff members focus on judging the five evaluation indicators on the transformation of the alternatives, and the judgment matrix is structured as follows:

Tab.1 decision maker D_1 (technical staff)

	U_1	U_2	U_3	U_4	U_5
x_1	PF	PF	F	G	VG
x_2	VG	PF	VG	F	G
x_3	VG	G	PF	VG	F
x_4	G	G	E	VG	G

Tab.2 decision maker D_2 (manager)

	U_1	U_2	U_3	U_4	U_5
x_1	VG	G	VG	PF	F
x_2	G	G	F	VG	VG
x_3	G	VG	F	G	G
x_4	PF	F	VG	PF	PF

Tab.3 decision maker D_3 (production staff)

	U_1	U_2	U_3	U_4	U_5
x_1	G	F	PF	E	G
x_2	F	VG	G	G	E
x_3	PF	E	G	E	VG
x_4	VG	VG	E	G	E

;
$$B=\begin{bmatrix}1 & 5 & 3 & 2 & 5\\ \frac{1}{5} & 1 & 2 & 4 & 3\\ \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{3} & 2\\ \frac{1}{2} & \frac{1}{4} & 3 & 1 & 2\\ \frac{1}{5} & \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & 1\end{bmatrix} \quad (2)$$

According to the matrix eigen value simple algorithm to be:

$$B^*=\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 1\end{bmatrix} \quad (3)$$

Because of the existence of $B^*\circ B^*\subseteq B^*$, it can get corresponding weights of these five indicators $w_1=\sqrt[5]{1\times5\times3\times2\times5}$ =2.724, by the same token: w_2=1.369; w_3=0.6444; w_4=1.059; w_5=0.4409. So the index weight of FAHP is:

$$w=(2.724,1.369,0.6444,1.059,0.4409)^T \quad (4)$$

Step2: Normalize the index weight of FAHP $w=(2.724,1.369,0.6444,1.059,0.4409)^T$ into $w_j=(0.4367,0.2195,0.1033,0.1698,0.0707)^T$, where $\sum_{j=1}^{5} w_j$ =1. Use GIOWA operator to accumulate the linguistic evaluation information in the i line of the matrix R_k to get the comprehensive attribute evaluation $r_i^{(k)}$ (i=1,2,3,4;k=1,2,3)of the decision scheme x_i by decision maker D_k. As a result, it can obtain the comprehensive attribute evaluation information according to each scheme fixed by decision maker D_1. $r_{11}^{(1)}$= PF, $r_{12}^{(1)}$= PF, $r_{13}^{(1)}$= F, $r_{14}^{(1)}$=G, $r_{15}^{(1)}$=VG, in addition, S=(excellent, perfect, very good, good, fair, poor, very poor, awful, extremely poor).,so $r_{11}^{(1)}\approx r_{12}^{(1)}$ =PF > $r_{15}^{(1)}$ =VG > $r_{14}^{(1)}$ =G > $r_{13}^{(1)}$ =F.

According to the given linguistic scales, it can get triangular fuzzy numbers:

$\hat{a}_{11}^{(1)}$ =[0.7,0.8,0.9]; $\hat{a}_{12}^{(1)}$ =[0.7,0.8,0.9]; $\hat{a}_{13}^{(3)}$ =[0.4,0.5,0.6]; $\hat{a}_{14}^{(1)}$ =[0.5,0.6,0.7]; $\hat{a}_{15}^{(1)}$ =[0.6,0.7,0.8], corresponding to the sum $r_{1j}^{(1)}$ ($j=1,2,3,4,5$), thus $b_{11}^{(1)}=b_{12}^{(1)}=\hat{a}_{11}^{(1)}=\hat{a}_{12}^{(1)}=$ [0.7,0.8, 0.9]; $b_{13}^{(1)}=\hat{a}_{15}^{(1)}$=[0.6,0.7,0.8]; $b_{14}^{(1)}=\hat{a}_{14}^{(1)}$= [0.5,0.6,0.7]; $b_{15}^{(1)}=\hat{a}_{13}^{(3)}$= [0.4,0.5,0.6].

Then the programs on the decision making preferences of D_1 are as follows:

$z_1^{(1)}(w)$=0.8345; Simultaneously, $z_2^{(1)}(w)=z_3^{(1)}(w)$=0.81255; $z_4^{(1)}(w)$=0.85296

By the same token, the programs on the decision making preferences of D_2 can be:

$z_1^{(2)}(w)$=0.81255; $z_2^{(2)}(w)$=0.75855; $z_3^{(2)}(w)$=0.7366; $z_4^{(2)}(w)$=0.86181

And D_3:

$z_1^{(3)}(w)$=0.86784; $z_2^{(3)}(w)$=0.84589; $z_3^{(3)}(w)$=0.9345; $z_4^{(3)}(w)$=0.92417

Step3: Assume that the weight of decision makers is $w^*=(0.3,0.3,0.4)^T$, accumulate the comprehensive attribute evaluation $z_i^{(k)}(w)$ (k=1,2,3)of the programs x_i given by these three categories decision makers through the GIOWA operator, carry out the comprehensive group attribute evaluation $z_i(w^*)$, $(i=1,2,\cdots,n)$:

$x_1=z_1(w^*)$=1.0391

Similar results can be obtain: $x_2=z_2(w^*)$=0.8010; $x_3=z_3(w^*)$=0.8188; $x_4=z_4(w^*)$=0.8770.

Step4: Sort all the programs: $x_1 > x_4 > x_3 > x_2$, so the best decision is x_1.

If it is calculated in accordance with the literature [5] after step 2, there will be a different result, especially when decision making programs are more than 5 and the decision making strength appears not to be enough, There may exist the same results between multiple options. However, this method can be applied to a specific sort of decision optimization.

And the order of these programs is: $x_4 > x_1 \approx x_2 \approx x_3$, among which x_4 is the best decision.

Conclusion

By discussion, the author proposes a FAHP simple solution to test the consistency in AHP, which provides a new train of thought for the expert group decision making; FAG model can provide an accurate sequencing optimization in decision making process, especially when the decisions are more than five. And the result is not based on the GIOWA fuzzy linguistic decision making which may not get satisfactory result because of the existence of multiple results while it is bound to use another ways to exclude decisions, which will increase the complexity of decision making; It is more ideal to solve the problem of the weight distribution of the attribute and the sequence of linguistic decision making.

References

[1] T L. Satty. The Analytic Hierarchy Process[M].New York：McGrow—Hill,1980.

[2] Z.S Xu, Q.L Da. An Overview of Operators for Aggregating Information [J]. International Journal of Intelligent Systems,2003,(9)： 953～969.

[3] Z.S Xu. Method Based on Fuzzy Linguistic Assessments And Giowa Operator in Multi-Attribute Group Decision-Making [J]. Journal of Systems Science and Mathematical Sciences,2004,(4)：218～224.

[4] Shui-li Chen,Jing-gong Li,Xiang-gong Wang. Fuzzy set theory and its applications[M]. Beijing: Science Press,2005,242～246. (in Chinese)

[5] Z.S Xu. Uncertain multi-attribute decision-making method and its application [M]. Beijing: Tsinghua University Press,2004,170.(in Chinese)