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On Quantum Systems of Particles with Singular Magnetic Interaction

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Abstract

For systems of particles with singular magnetic interation for special choice of a selfadjoint extension of the Hamiltoniam equilibrium reduced density matrices are calculated in the thermodynamic limit for simplest pair magnetic potentials.

The quantum system of n particles with magnetic interaction can be defined by the Hamiltonian of a system of n ν-d charged particles moving in the electromagnetic collective field characterized by the vector potential $A_j(X_n)$ that depends on the position vectors $X_n = (x_1, ..., x_n)$ from νn-dimensional space. It is reasonable to assume that A_j is singular on the set of coinciding position vectors of the particles and that the singularity is not stronger than the distance between particles to some negative finite power. Then the initial Hamiltonian $\dot{H}_n$ as a symmetric operator defined on $C_0(\mathbf{R}_0^{\nu n}), \mathbf{R}_0^{\nu n} = \mathbf{R}^{\nu n} \setminus \bigcup_{k<j}(x_j = x_k)$, is given by

$$\dot{H}_n = \frac{1}{2}\sum_{j=1}^{n}(p_j - A_j(X_n))^2,$$

$$p_j = i^{-1}\partial_j, \ \ (p_j - A_j)^2 = \sum_{\alpha=1}^{\nu}(p_j^\alpha - A_j^\alpha)^2,$$

and ∂_j is a partial derivative with recpect to x_j. The motivation to study such systems appeared recently when it was realized that one of them (CS-system) can be derived in 3-d topological or Chern–Simons (CS) nonrelativistic electrodynamics.Vector potentials A_j for it are given by the skew partial derivative with respect to x_j of the Coulomb potential energy of a system of n charged particles [1]. It is believed that the system with Fermi statistics can describe phenomena of high-temperature superconductivity [2–3] and there are phase transitions [4–5]. The nonlinear Schrödinger equation obtained in the mean-field limit has soliton solutions [1] and there is a hidden SO(2,1) symmetry for a CS particle system.

There is a nontrivial problem of description of equilibrium states for such systems in the thermodynamic limit for the case

$$A_j(X_n) = \sum_{k\in(1,..,j-1,j+1,...n)} a_j(x_j - x_k).$$

For such classical systems, the grand partition function coincide with that of a free particle system but the Gibbs (grand canonical) correlation functions are computed easily only in the case of short-range interactions(a_j is an integrable function). The existence of the functions for long-range magnetic interactions (a_j are not integrable functions) is an open problem (CS-interaction is long-range). We showed [6] that for the classical CS-system in the mean-field type limit (the thermodynamic limit is performed simultaneously) the correlation functions converge to functions depending on momenta of particles only, which do no factorize into a product of one-particle correlation functions. In quantum case the situation is more complicated. Up to now there are no results concerning an existence of reduced density matrices (RDMs) even for short-range magnetic interaction with a general pair vector magnetic potential a_j. Substantial simplification is achieved if

$$A_j(X_n) = \partial_j U(X_n),$$

$$X_n \in \mathbf{R}_0^n,$$

$$U(X_n) = \sum_{1\le k<j\le n} \phi(x_j - x_k).$$

It is remarkable that the CS-interaction allows the representation for its vector potential with $\phi(x) = \phi_{CS}(x) = \arctan \frac{x^2}{x^1}, \quad x = (x^1, x^2)$. For such systems (almost integrable), the following equality is true

$$\dot{H}_n = \exp\{i\hat{U}\}\dot{H}_n^0 \exp\{-i\hat{U}\},$$

where $\hat{U}$ is the operator of multiplication by $U(X_n)$, $\dot{H}_n^0 = -\frac{1}{2}\dot{\Delta}_n$, $\dot{\Delta}_n$ is the νn-dimensional Laplacian restricted to $C_0^\infty(\mathbf{R}_0^n)$.

It is obvious that there exists the following operator H_n with the domain

$$D(H_n) = \exp\{i\hat{U}\}D(\Delta_n)$$

which is the self-adjoint extension of $\dot{H}_n$

$$H_n = \exp\{i\hat{U}\}H_n^0 \exp\{-i\hat{U}\},$$

$$H_n^0 = -\frac{1}{2}\Delta_n.$$

For this extension a grand partition function for Maxwell–Boltzmann (MB) statistics coincides with that of a free particle system. Moreover, if the magnetic potential is short-range, the RDMs for a Dirichlet boundary condition are computed easily in the thermodynamic limit [7]. For long-range magnetic interactions it is difficult to prove the existence of RDMs in the thermodynamic limit. But in one dimension, there is an exceptional system for which RDMs can be found in this limit for MB statistics. It is defined by U expressed as a Coulomb potential energy of a system of n charged particles. It was established by us that RDMs are nontrivial in the thermodynamic limit if the differences of variables sit on a lattice [7]. This system can be considered as a one-dimensional analog of the CS-system. We confirm this result for this one-dimensional system with Fermi and Bose statistics for the above self-adjoint extension and Dirichlet boundary condition [9]. We also find the expression for RDMs for the ν-dimensional system in the thermodynamic limit for a short-range magnetic potential, adapting the technique worked out by Ginibre [8] for usual systems of particles interacting via a pair potential. Of course, there are other self-adjoint extensions of $\dot{H}_n$ and there is a problem of their description. The treatment of the problem for 1-d systems and its relation to 1-d anyons can be found in [7]. The fact that the RDMS $\rho(X_n|Y_n)$ in the thermodynamic limit for an 1-d analog of the CS-system are nontrivial if $x_j - y_j$ belong to the 1-d lattice is very surprising. Is this discretization true for all the systems with long-range magnetic interaction(a_j are not integrable functions)?

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