Nonlinear Mathematical Physics 1997, V.4, N 1–2, 231–240.

Sound Attenuation in a Circular Duct of a Viscous Medium in the Absence of Mean Flow

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Submitted by W. FUSHCHYCH
Received November 25, 1995

Abstract

An analytical method to study the effect of viscosity of a medium and the wave number on sound propagation and sound attenuation numbers in circular ducts has been presented. The method is based on the variation of parameters of the solution corresponding to the case of inviscid acoustic waves in circular ducts and axisymmetric modes. A mathematical model is constructed to describe the physical problem in general. Three basic assumptions have been considered, namely, each flow quantity has been written as the sum of a steady mean flow and an unsteady acoustic flow quantity. The effect of thermal conductivity of the gas has been neglected as well as no mean flow. The results for a wide range of wave numbers and Reynolds numbers show that for a viscous medium, the propagation number is a weak function of the Reynolds number, and as the Reynolds number increases, the propagation number approaches its inviscid value. Also the propagation number is independent of the wave number. For the attenuation number, it decreases monotonically with the increase of the Reynolds number and it vanishes when Reynolds number exceeds 10^4.

1 Introduction

Study of the attenuation of sound propagation in ducts with sound-absorbing material on the interior of the walls has received considerable attention from researchers since nearly the middle of the twentieth century. In 1939, Morse [8] studied rigorously the propagation of sound in ducts lined with absorbing material, but his studies were limited to the case of liners in which axial wave proparation does not occur. Scott [15], in 1946, took into consideration axial wave propagation in the liner. Due to the importance of viscosity of the medium in the field of duct acoustics, Rayleigh [13] in 1945 studied its effect on the attenuation of acoustic modes in ducts. In fact, the viscosity of the medium together with its thermal conductivity are partially responsible for the natural attenuation of sound waves. The effect of these two parameters had been studied experimentally during the early years of the 1950's by Beatty [1] in 1950, Shaw [16] in 1953 and Lambert [2] in 1953. During the 1960's and early 1970's, the need to reduce aircraft engine noise became essential and of major concern to the public and governments, and the researchers directed their attention to study the duct acoustic problem in the presence of mean flow in the duct. The earliest such papers appeared to be that of Pridmore–Brown [12] in 1958, who considered the propagation of sound in a flat duct when the velocity profile was given by the one seventh power law. The same problem, for the case of hard-walled duct, was solved numerically by Mungur and Gladwell [9] in 1969. Their numerical calculations were based upon a Runge–Kutta algorithm. In the same year, Mungur and Plumblee [10] extended the calculations of Mungur and Gladwell [9] to a soft-walled annulus. In 1973, Mikhail [3] and Mikhail and Abdelhamid [4–6] studied sound propagation in ducts, with finite admittance at the walls, in the presence of viscous mean flow.

The interesting paper by Nayfeh, Kaiser and Telionis [11], in 1975, refers to more than 150 references as well as reviews all aspects of duct acoustics.

In 1993, Mikhail and Tantawy [7] studied the effect of the medium viscosity on sound propagation and attenuation in two-dimensional ducts in the absence of mean flow.

Analytic studies for the case of sound propagation for general mean flows in tunnels with a viscous medium have not been found.

In this paper we study analytically the effect of viscosity of the medium and the wave number, in the absence of mean flow, on the propagation number and the attenuation number.

In fact, our work is a necessary preliminary to the discussion of the more general problem of attenuation of sound in tunnels of a viscous and moving medium.

2 Formulation of the problem

2.1 Basic equations

The flow field inside a circular duct given by Schlichting [14], is governed by the following five basic equations in cylindrical coordinates.

(1) Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0, \tag{2.1}$$

where ρ is the density, t is the time, (v_r, v_θ, v_z) are velocities in the (r, θ, z) directions, respectively.

(2) Momentum equation:

r-component

$$\begin{aligned} &\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z}\right) = \\ &\qquad -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(2\mu r\frac{\partial v_r}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r}\right)\right] + \\ &\qquad \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)\right] - \frac{2}{3r}\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_\theta}{\partial \theta} + r\frac{\partial v_z}{\partial z}\right)\right], \end{aligned} \tag{2.2}$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}\right)=$$
$$-\frac{1}{r}\frac{\partial p}{\partial \theta}+\frac{1}{r}\frac{\partial}{\partial r}\left[\mu\left(r\frac{\partial v_\theta}{\partial r}+\frac{\partial v_r}{\partial \theta}-v_\theta\right)\right]+\frac{2}{r^2}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{\partial v_\theta}{\partial \theta}+v_r\right)\right]+ \tag{2.3}$$
$$\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{r\partial \theta}+\frac{\partial v_\theta}{\partial z}\right)\right]-\frac{2}{3r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}\right)\right],$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=$$
$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right]+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta}+\frac{\partial v_\theta}{\partial z}\right)\right]+ \tag{2.4}$$
$$\frac{\partial}{\partial z}\left(2\mu\frac{\partial v_z}{\partial z}\right)-\frac{2}{3}\frac{\partial}{\partial z}\left\{\mu\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}\right]\right\}.$$

(3) Energy equation for a perfect gas:

$$\rho c_p\frac{\partial T}{\partial t}=\frac{\partial p}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{k}{r}\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\mu\phi_{00}, \tag{2.5}$$

where k is the thermal conductivity of a gas, c_p is the specific heat at constant pressure, T is the temperature, and ϕ_{00} is the dissipation function given by:

$$\phi_{00}=2\left[\left(\frac{\partial v_r}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right)^2+\left(\frac{\partial v_z}{\partial z}\right)^2\right]+\left(\frac{\partial v_\theta}{\partial r}+\frac{1}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta}{r}\right)^2+$$
$$\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right)^2+\left(\frac{\partial v_\theta}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2-\frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}\right]^2.$$

(4) State equation

$$P=\rho RT, \tag{2.6}$$

where R is the gas specific constant.
Boundary condition:
Along the boundary of a duct, the radial component of a velocity vanishes.

2.2 Perturbation of the flow

Following Nayfeh, Kaiser and Telionis [11], we assume that each flow quantity $q(r,t)$, which corresponds to ρ, u, v, p, T and ϕ_{00}, is the sum of a steady mean flow $q_0(t)$ and an unsteady acoustic flow quantity $q_1(r,t)$, where r is the position vector. Also, in order to focus on the effect of viscosity on the attenuation of acoustic waves, we neglect the effect of thermal conductivity of the gas, i.e., we set $k=0$. Then, substituting the assumed expressions of the flow quantities into equations (2.1) to (2.6), taking into consideration that the steady mean flow quantities satisfy the basic equations and neglecting nonlinear acoustic quantities, we obtain

$$\frac{\partial \rho_1}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho_0 r u_{1r}+\rho_1 r u_{0r})+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho_0 u_{1\theta}+\rho_1 u_{0\theta})+\frac{\partial}{\partial z}(\rho_0 u_{1z}+\rho_1 u_{0z})=0, \tag{2.7}$$

$$\begin{gathered}\rho_1\left(\frac{\partial u_{0r}}{\partial t}+u_{0r}\frac{\partial u_{0r}}{\partial r}+\frac{u_{0\theta}}{r}\frac{\partial u_{0\theta}}{\partial r}+u_{0z}\frac{\partial u_{0r}}{\partial z}\right)+\\ \rho_0\left(\frac{\partial u_{1r}}{\partial t}+u_{1r}\frac{\partial u_{0r}}{\partial r}+\frac{u_{1\theta}}{r}\frac{\partial u_{0r}}{\partial r}+\frac{u_{0\theta}}{r}\frac{\partial u_{1r}}{\partial r}+u_{1z}\frac{\partial u_{0r}}{\partial r}\right)=\\ -\frac{\partial p_1}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}\left(2\mu r\frac{\partial u_{1r}}{\partial r}\right)-\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{\partial u_{1\theta}}{\partial r}+\frac{1}{r}\frac{\partial u_{1r}}{\partial \theta}-\frac{u_{1\theta}}{r}\right)\right]+\\ \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_{1z}}{\partial r}+\frac{\partial u_{1r}}{\partial z}\right)\right]-\frac{2}{3r}\frac{\partial}{\partial r}\left[\mu r\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_{1r})+\frac{1}{r}\frac{\partial u_{1\theta}}{\partial \theta}+\frac{\partial u_{1z}}{\partial z}\right)\right],\end{gathered} \tag{2.8}$$

$$\begin{gathered}\rho_1\left(\frac{\partial u_{0\theta}}{\partial t}+u_{0r}\frac{\partial u_{0\theta}}{\partial r}+\frac{u_{0\theta}}{r}\frac{\partial u_{0\theta}}{\partial \theta}+u_{0z}\frac{\partial u_{0\theta}}{\partial z}\right)+\\ \rho_0\left(\frac{\partial u_{1\theta}}{\partial t}+u_{1r}\frac{\partial u_{0\theta}}{\partial r}+u_{0r}\frac{\partial u_{1\theta}}{\partial r}+\frac{u_{1\theta}}{r}\frac{\partial u_{0\theta}}{\partial \theta}+\frac{u_{0\theta}}{r}\frac{\partial u_{1\theta}}{\partial \theta}+u_{1z}\frac{\partial u_{0\theta}}{\partial z}+u_{0z}\frac{\partial u_{1\theta}}{\partial z}\right)\\ =-\frac{1}{r}\frac{\partial p_1}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}\left[\mu r\left(\frac{\partial u_{1\theta}}{\partial r}+\frac{1}{r}\frac{\partial u_{1r}}{\partial \theta}-\frac{u_{1\theta}}{r}\right)\right]+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{\partial u_{1z}}{\partial \theta}+\frac{\partial u_{1\theta}}{\partial z}\right)\right]+\\ \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_{1z}}{r\partial \theta}+\frac{\partial u_{1\theta}}{\partial z}\right)\right]-\frac{2}{3r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_{1r})+\frac{1}{r}\frac{\partial u_{1\theta}}{\partial \theta}+\frac{\partial u_{1z}}{\partial z}\right)\right].\end{gathered} \tag{2.9}$$

$$\begin{gathered}\rho_1\left(\frac{\partial u_{0z}}{\partial t}+u_{0r}\frac{\partial u_{0z}}{\partial r}+\frac{u_{0\theta}}{r}\frac{\partial u_{0z}}{\partial \theta}+u_{0z}\frac{\partial u_{0z}}{\partial z}\right)+\\ \rho_0\left(\frac{\partial u_{1z}}{\partial t}+u_{1r}\frac{\partial u_{0z}}{\partial r}+u_{0r}\frac{\partial u_{1z}}{\partial r}+\frac{u_{1\theta}}{r}\frac{\partial u_{0z}}{\partial r}+\frac{u_{0\theta}}{r}\frac{\partial u_{1z}}{\partial r}+u_{1z}\frac{\partial u_{0z}}{\partial z}+u_{0z}\frac{\partial u_{1z}}{\partial z}\right)\\ =-\frac{\partial p_1}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[\mu r\left(\frac{\partial u_{1z}}{\partial r}+\frac{\partial u_{1r}}{\partial z}\right)\right]+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial u_{1z}}{\partial \theta}+\frac{\partial u_{1\theta}}{\partial z}\right)\right]+\\ \frac{\partial}{\partial z}\left(2\mu\frac{\partial u_{1z}}{\partial z}\right)-\frac{2}{3}\frac{\partial}{\partial z}\left[\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_{1r})+\frac{1}{r}\frac{\partial u_{1\theta}}{\partial \theta}+\frac{\partial u_{1z}}{\partial z}\right)\right],\end{gathered} \tag{2.10}$$

$$\rho_0 c_p\frac{\partial T_1}{\partial t}+\rho_1 c_p\frac{\partial T_0}{\partial t}=\frac{\partial p_1}{\partial t}+\mu\phi_{1,00}, \tag{2.11}$$

$$p_1=\rho_0 R T_1+\rho_1 R T_0. \tag{2.12}$$

The solution of each flow quantity $q(r,t)$ is assumed to be of the form

$$q(r,t)=q(r)\exp(i(\omega t+m\theta)-\gamma z), \tag{2.13}$$

where ω is the acoustic frequency, γ is the complex wave number, and m is an integer. Substituting (2.13) into equations (2.7) to (2.12), noting that $u_{0r}=0$, $u_{0\theta}=0$, $u_{0z}=V(r)$, and $m=0$ which corresponds to axisymmetric modes, we get

$$\Omega\rho_1=\rho_0\left(\gamma u_z-\frac{du_z}{dr}-\frac{u_r}{r}\right), \tag{2.14}$$

where $\Omega = i\omega - \gamma V(r)$

$$\frac{4\mu}{3}\frac{d^2u_r}{dr^2} + \frac{2\mu}{3r}\frac{du_r}{dr} + (\mu\gamma^2 - \rho_0\Omega)u_r = \frac{dp}{dr} + \frac{\gamma\mu}{3}\frac{du_z}{dr} - \frac{2\mu\gamma}{3}\frac{u_z}{r}, \tag{2.15}$$

$$(i\omega\rho_0 - \gamma V)u_\theta - \frac{\mu}{r}\left(r\frac{d^2u_\theta}{dr^2} + \frac{du_\theta}{dr}\right) - \mu\gamma^2 u_\theta = 0, \tag{2.16}$$

$$\mu\frac{d^2u_z}{dr^2} + \frac{\mu}{r}\frac{du_z}{dr} + \left(\frac{4}{3}\mu\gamma^2 - \rho_0\Omega\right)u_z = \frac{\mu\gamma}{3}\frac{du_r}{dr} + \left(\rho_0\frac{dV}{dr} + \frac{\mu\gamma}{3r}\right)u_r - \gamma p, \tag{2.17}$$

$$i\omega T = \frac{i\omega p}{\rho_0 c_p} + \frac{2\mu}{\rho_0 c_p}\frac{dV}{dr}\left(\frac{du_z}{dr} - \gamma u_r\right), \tag{2.18}$$

$$p = R(\rho_0 T + T_0\rho). \tag{2.19}$$

2.3 Normalized perturbed governing equations

Velocities, lengths, time, density, pressure, and temperature, can be made dimensionless by using the ambient speed of sound c_a, a characteristic duct radius r_a, the time r_a/c_a, the density ρ_a, the ambient pressure $\rho_a c_a^2$, and the mean temperature T_0, respectively. Denote these dimensionless variables by $u_n, v_n; r_n, z_n; t_n, \rho_n, p_n$, and T_n. Substituting the normalized parameters in equations (2.14) to (2.19), after dropping the subscripts n and 1 we get the following governing equations

$$\Omega\rho = \rho\left(\delta u_z - \frac{du_r}{dr} - \frac{u_r}{r}\right), \tag{2.20}$$

$$\frac{1}{Re}\left(\frac{d^2u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr}\right) + \left(\frac{4}{3}\frac{\delta^2}{Re} - \rho_0\Omega\right)u_z = \frac{\delta}{3Re}\left(\frac{du_r}{dr} + \frac{u_r}{r}\right) + \rho_0 u_r\frac{dM}{dr} - \delta p, \tag{2.21}$$

$$\frac{4}{3Re}\frac{d^2u_r}{dr^2} + \frac{2}{3rRe}\frac{du_r}{dr} + \left(\frac{\delta^2}{Re} - \rho_0 z\right)u_r = \frac{\delta}{3Re}\left(\frac{du_z}{dr} - 2\frac{u_z}{r}\right) + \frac{dp}{dr}, \tag{2.22}$$

$$(\Omega + \delta M)\frac{T\rho_0}{\nu - 1} = (\Omega + \delta M)p + \frac{2}{Re}\frac{dM}{dr}\left(\frac{du_z}{dr} - \delta u_r\right), \tag{2.23}$$

$$p = \frac{1}{\nu}(\rho_0 T + \rho), \tag{2.24}$$

where $\Omega(r) = i\omega(1 - kM)$, $k = \dfrac{\gamma}{iK}$, $K = \dfrac{\omega}{c_a}$, $\gamma r_a = \delta = iKkr_a = \delta_1 + i\delta_2$, and $Re = \dfrac{\rho_a c_a r_a}{\mu}$ is the Reynolds number.

Eliminating ρ and T from equations (2.20) to (2.24), we get three equations in the three unknowns p, u_r and u_z which are:

$$\frac{1}{Re}\left(\frac{d^2u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr}\right) + \left(\frac{4}{3}\frac{\delta^2}{Re} - \rho_0\Omega\right)u_z = \frac{\delta}{3Re}\left(\frac{du_r}{dr} + \frac{u_r}{r}\right) + \rho_0 u_r\frac{dM}{dr} - \delta p, \tag{2.25}$$

$$\frac{4}{3Re}\frac{d^2u_r}{dr^2} + \frac{2}{3rRe}\frac{du_r}{dr} + \left(\frac{\delta^2}{Re} - \rho_0\Omega\right)u_r = \frac{dp}{dr} + \frac{\delta}{3Re}\left(\frac{du_z}{dr} - \frac{2u_z}{r}\right), \tag{2.26}$$

$$p(\Omega + \delta M) = \frac{2(\nu - 1)}{Re}\frac{dM}{dr}\left(\frac{du_z}{dr} - \delta u_r\right) + \rho_0\left(1 + \frac{\delta M}{\Omega}\right)\left(\delta u_z - \frac{du_r}{dr} - \frac{u_r}{r}\right). \tag{2.27}$$

3 The case of inviscid acoustic waves with a zero mean flow

In the case of inviscid acoustic waves, we substitute $\mu = 0$ or equivalently $Re \to \infty$. Hence, equations (2.25) to (2.27) take the form

$$\rho_0 \Omega u_z = \delta p - \rho_0 u_r \frac{dM}{dr}, \tag{3.1}$$

$$\rho_0 \Omega u_r = -\frac{dp}{dr}, \quad \text{and} \tag{3.2}$$

$$p(\Omega + \delta M) = \rho_0 \left(1 + \frac{\delta M}{\Omega}\right)\left(\delta u_z - \frac{du_r}{dr} - \frac{u_r}{r}\right).$$

Eliminating u_r and u_z from these three equations, we get the following equation for pressure

$$\frac{d^2p}{dr^2} + \frac{1}{r}\frac{dp}{dr} + \frac{2k}{1-kM}\frac{dM}{dr}\frac{dp}{dr} + r_a^2 K^2[(1-kM)^2 - k^2]p = 0.$$

If we assume no mean flow , i.e., $M = 0$, then the equation for the pressure takes the form:

$$\frac{d^2p}{dr^2} + \frac{1}{r}\frac{dp}{dr} + r_a^2 K^2 (1-k^2)p = 0. \tag{3.3}$$

But

$$r_a^2 K^2 (1-k^2) = r_a^2 K^2 \left[1 - \left(\frac{\gamma c_a}{i\omega}\right)^2\right] = r_a^2 K^2 \left(1 + \frac{\gamma^2 c_a^2}{\omega^2}\right) > 0.$$

Hence, equation (3.3) represents the zeroth-order Bessel differential equation which has the general solution

$$p(r) = AJ_0\left(r_a K\sqrt{1-k^2}\, r\right) + BY_0\left(r_a K\sqrt{1-k^2}\, r\right), \tag{3.4}$$

and on the basis of physical considerations, we take $B = 0$.

At $r = 1, u_r = 0$, hence from equation (3.2) we get $\dfrac{dp}{dr} = 0$, and, consequently, by using (3.4), we get

$$Ar_a\sqrt{1-k^2}J_1\left(r_a K\sqrt{1-k^2}\right) = 0,$$

where $r_a K\sqrt{1-k^2} = A_n$ are the roots of $J_1(x) = 0$.

Hence,

$$\gamma^2 = \frac{A_n^2}{r_a^2} - \frac{\omega^2}{c_a^2}.$$

Note that no propagation occurs unless the frequency is higner than the cut-off frequency, i.e., γ is imaginary. The condition for propagation is then

$$\omega > \frac{A_n c_a}{r_a}.$$

From (3.1), (3.4) and $M = 0$ we have

$$u_z = \frac{\delta A}{\rho_0 \Omega} J_0 \left(r_a K \sqrt{1-k^2}\, r \right),$$

and from (3.2), (3.4) and $M = 0$ we have

$$u_r = \frac{A r_a K \sqrt{1-k^2}}{\rho_0 \Omega} J_1 \left(r_a K \sqrt{1-k^2}\, r \right).$$

For the case of $x = \left(r_a K \sqrt{1-k^2}\, r \right) \gg 1$, we can approximate

$$J_0(x) = \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} \right) + o\left(\frac{1}{x} \right).$$

Hence,

$$u_z = A_0 r^{-1/2} \cos\left(ar - \frac{\pi}{4} \right), \qquad \text{and}$$

$$u_r = A_1 r^{-1/2} \sin\left(ar - \frac{\pi}{4} \right) + A_2 r^{-3/2} \cos\left(ar - \frac{\pi}{4} \right),$$

where

$$A_0 = \sqrt{\frac{2}{\pi a}}\, \frac{\delta A}{\rho_0 \Omega},\ A_1 = \sqrt{\frac{2a}{\pi}}\, \frac{A}{\rho_0 \Omega},\ A_2 = \sqrt{\frac{2}{\pi a}}\, \frac{A}{2\rho_0 \Omega},\ a = r_a K \sqrt{1-k^2}.$$

4 Case of viscous acoustic waves with a zero mean flow

In this case, we have $M = M_0 = 0,\ \rho_0 = 1,\ T_0 = 1$, hence $\Omega = i r_a K$. Therefore, equations (2.25) to (2.27) become

$$\frac{1}{Re}\left(\frac{d^2 u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr} \right) + \left(\frac{4}{3}\frac{\delta^2}{Re} - \Omega \right) u_z = \frac{\delta}{3Re}\left(\frac{du_r}{dr} + \frac{u_r}{r} \right) - \delta p, \tag{4.1}$$

$$\frac{4}{3Re}\frac{d^2 u_r}{dr^2} + \frac{2}{3rRe}\frac{du_r}{dr} + \left(\frac{\delta^2}{Re} - \Omega \right) u_r = \frac{dp}{dr} + \frac{\delta}{3Re}\left(\frac{du_z}{dr} - \frac{2u_z}{r} \right), \tag{4.2}$$

$$\Omega p = \delta u_z - \frac{du_r}{dr} - \frac{u_r}{r}. \tag{4.3}$$

Eliminating the pressure p from the three equations (4.1), (4.2) and (4.3), we get two simultaneous equations in u_r and u_z as follows

$$\frac{d^2 u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr} + R_1 u_z - R_z\left(\frac{du_r}{dr} + \frac{u_r}{r} \right) = 0, \qquad \text{and} \tag{4.4}$$

$$R_3 \frac{d^2 u_r}{dr^2} + \left(R_3 - \frac{1}{2} \right)\frac{1}{r}\frac{du_r}{dr} + R_4 u_r - R_5 \frac{u_r}{r^2} - \frac{3}{4} R_2 \frac{du_z}{dr} + R_6 \frac{u_z}{r} = 0, \tag{4.5}$$

where

$$R_1 = \frac{4}{3}\delta^2 - \Omega Re + \frac{\delta^2 Re}{\Omega}, \quad R_2 = \frac{\delta}{3} + \frac{\delta Re}{\Omega}, \quad R_3 = 1 + \frac{3Re}{4\Omega},$$
$$R_4 = \frac{3}{4}\delta^2 - \frac{3\Omega}{4}Re, \quad R_5 = \frac{3Re}{4\Omega}, \quad R_6 = \frac{\delta}{2}.$$

We postulate a solution of the form

$$u_z = A_0 r^{-1/2} \cos\left(ar - \frac{\pi}{4}\right), \qquad \text{and} \tag{4.6}$$

$$u_r = A_1 r^{-1/2} \sin\left(ar - \frac{\pi}{4}\right) + A_2 r^{-3/2} \cos\left(ar - \frac{\pi}{4}\right) \tag{4.7}$$

where A_0, A_1, and A_2 are functions of Re, a, and δ. Substituting (4.6) and (4.7) into (4.4) and (4.5), we get

$$B_1 r^{-3/2} \sin\left(ar - \frac{\pi}{4}\right) + \left(B_2 r^{-1/2} + B_3 r^{-5/2}\right) \cos\left(ar - \frac{\pi}{4}\right) = 0, \tag{4.8}$$

and

$$\left(B_4 r^{-1/2} + B_5 r^{-5/2}\right) \sin\left(ar - \frac{\pi}{4}\right) + B_6 r^{-3/2} \cos\left(ar - \frac{\pi}{4}\right) = 0, \tag{4.9}$$

where

$$\left.\begin{aligned}
B_1 &= R_2\left(aA_2 - \frac{A_1}{2}\right),\\
B_2 &= A_0\left(R_1 - a^2\right) - R_2 a A_1,\\
B_3 &= \frac{A_0}{4} + R_2\frac{A_2}{2},\\
B_4 &= A_1\left(R_4 - R_3 a^2\right) + \frac{3}{4}R_2 a A_0,\\
B_5 &= A_1\left(\frac{R_3}{4} - R_5 + \frac{1}{4}\right) + A_2\left(2R_3 a + \frac{a}{2}\right),\\
B_6 &= A_1\frac{a}{2} + A_0\left(R_6 + \frac{3}{8}R_2\right) + A_2(R_4 - R_3 a^2).
\end{aligned}\right\} \tag{4.10}$$

From (4.8) and (4.9)

$$B_1 B_6 - r^2 B_2 B_4 = 0$$

from which we get

$$B_2 B_4 = 0, \qquad \text{and} \qquad B_1 B_6 = 0.$$

Assume

$$\frac{A_2}{A_1} = \frac{1}{2a}. \tag{4.11}$$

From the boundary condition $u_r = 0$ at $r = 1$ we get

$$\tan\left(a - \frac{\pi}{4}\right) = -\frac{A_2}{A_1}. \tag{4.12}$$

If $B_2 = 0$, then from (4.10)

$$A_1 = \frac{R_1 - a^2}{R_2 a} A_0. \tag{4.13}$$

If $B_6 = 0$, then from (4.10),

$$A_2 = \frac{R_1 - a^2 + 2R_6R_2 + \frac{3}{4}R_2^2}{2R_2(R_3a^2 - R_4)} A_0. \tag{4.14}$$

From (4.11) and (4.12) we get

$$a \tan\left(a - \frac{\pi}{4}\right) = -\frac{1}{2},$$

from which a can be determined. From (4.11), (4.13) and (4.14) we get

$$\left(R_1 - a^2 + 2R_2R_6 + \frac{3}{4}R_2^2\right) a^2 = (R_1 - a^2)(R_3a^2 - R_4),$$

from which δ and consequently δ_1 and δ_2 can be determined.

5 Results and discussion

We studied, analytically, the effect of viscosity of the medium and wave number r_aK on the attenuation number δ_1 and the propagation number δ_2 for a wide range of both large Reynolds numbers Re and wave numbers r_aK.

5.1 The effect of viscosity on the propagation number δ_2

From the analysis presented in section 3, it is shown that for an inviscid medium, the propagation constant δ_2 can be computed from the formula

$$\delta_2^* = \sqrt{(r_aK)^2 - A_n^2},$$

where A_n are the zeros of $J_1(x) = 0$.

We come to conclusion that the propagation number δ_2 is a weak function of Re, and that as Re increases, $\delta_2 \to \delta_2^*$ (its inviscid value). This result in in a good agreement with the results of Mikhail and Tantawy [7] for the case of ducts. Therefore, as is known, for all practical purposes we can neglect the effect of viscosity on the wave propagation.

Also the propagation number is dependent of r_aK.

5.2 The effect of viscosity on the attenuation number δ_1

On the study of effects of the medium viscosity on the attenuation number δ_1 for the zeroth, first and secondmodes, we found that: for the case of zero and higher modes, the attenuation of sound decreases monotonically with an increase of Re, and for high Re it decreases most rapidly by an increase of Re. Also, for $Re > 10^4$ the attenuation number tends to zero as Re increases. Therefore, for high Re, the attenuation is practically independent of the wave number r_aK.

For constant Re, we conclude that the attenuation number increases monotonically as the wave number, r_aK, increases.

In conclusion, we can say that the medium viscosity has a noticeable effect on the attenuation numbers and low Reynolds numbers. Therefore, in such situations we cannot neglect the viscosity of the medium.

Also, for fixed Re and r_aK the values of δ_1 of different modes vary slightly. This leads to the fact that the change in the sound wave is very slow along the circular duct length.

Dedication and Acknowledgement

The authoresses dedicate this work to their outstanding Professor, Dr. Radamees Bouthos Atiya, professor of acoustics, for his valuable contribution in this field.

Also the authoresses would like to acknowledge the referee for his critical review of the paper and valuable comments.

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