The Design For The Basic Rack Tooth Profile Of The Harmonic Gear Drive With Double-Circular-Arc

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Abstract. The normal section tooth profile of the hob basic rack is used creatively as the basic rack of the harmonic gear drive with double-circular-arc. Based on the clear selection methods of the basic rack tooth profile's main parameters, the optimization method and process of the basic rack tooth profile is proposed, and the validity and feasibility of the design is verified with example. By simulation, the composition of the circular spline (CS) tooth profile is determined, whose addendum and dedendum tooth profiles are formed by the flexible spline (FS) addendum tooth profile's enveloping, whose middle tooth profile is formed by the FS middle involute tooth profile's enveloping. It's concluded that the harmonic gear drive with this basic rack has fully realized the whole tooth conjugate contact, which is the most effective way to improve the comprehensive performance of the harmonic gear drive.

Introduction

Harmonic gear drive has high precision, large transmission ratio, light weight, small volume, powerful carrying capacity, high efficiency, and is easy to realize zero backlash, which is the key precision transmission mechanism in the robotic industry and precision positioning system, and has become the important basic parts of the modern industry[1].

Currently, the carrying capacity, transmission accuracy, operation life, weight and volume of the harmonic gear drive mainly depend on the meshing tooth profile, meshing parameters, FS structure and lubrication condition, and among them the meshing profile has the biggest influence. It's well known that in harmonic gear drive the involute tooth profile is mostly widely used, for its technology is very mature, the process is easy to implement, and batch production cost is very low, however, it has not fully realized conjugate transition. Even though, the involute tooth profile design can realize the continuous transmission between movement and power, the transmission is realized through the tooth profiles' edges contacting and sharp points meshing from the FS' s torsional deflection caused by load., this meshing state makes the gaps between the tooth profiles uneven, which hinders the formation of the liquid friction state, and makes meshing loss increase [2]. In order to solve these problems, based on the involute addendum modification principle the circular arc tooth profile [3], and 3C tooth profile[4] are proposed to replace the involute tooth profile. These profiles mentioned above are easy to form a wedge oil film with uniform backlash and improve the quality of the harmonic drive meshing, however, these profiles are mostly based on ideal models , hence difficult to manufacture.

Based on the four-roller wave generator in former Soviet Union the harmonic gear drive with double-circular-arc tooth profile was put forward[5], but the circular-arc and linear tooth profile was used in its CS,the CS and the FS were not fully conjugate transmission, the meshing area was relatively small, and the cutting tool's manufacturing process was complex. Based on the cosine cam wave generator in Japan the harmonic gear drive with S tooth profile was designed and developed[6], in which the circular-arc with bigger curvature replaced the original tooth profile in the addendum and dedendum. Compared with the original tooth profile, in the new design the bearing capacity was enhanced greatly. Due to the use of approximate method,at the beginning reduction ratio was small, and there was interference of tooth profiles in the harmonic reducer, which has been almost solved early this century. Leader drive has developed the harmonic gear transmission with P tooth profile, which has low tooth depth, but large meshing area, small FS deformation , and more meshing teeth, and has been widely used in ABB products.

The technical route different from S tooth profile of foreign countries is used, and the harmonic gear drive with double-circular-arc whose performance is as good as S tooth profile is proposed. The CS and FS are produced with the most mature processing technology of gear shaping and hobbing, the basic rack tooth profile is the normal section of the hob basic rack, and the harmonic gear drive with double-circular-arc is designed according to the harmonic gear drive meshing principle and the main processing characteristics.

The Design for the Basic Rack Tooth Profile of the Harmonic Drive with Double-Circular-Arc. The basic rack tooth profile is the normal section of the hob basic rack. The basic rack tooth profile must be designed on the following criteria: a) meshing area is larger than 30% and has high rigidity; b) there is no tooth profile overlapping interference and no edge contact in the conjugate tooth profiles; c) the machining tools' tooth profiles corresponding to the CS and FS's conjugate tooth profile are easy to process; d) the FS and CS has enough meshing depth; e) the FS addendum thickness to be processed is no less than 0.2 times the modulus; f) The tooth profile in meshing is the only consideration when designing, and the transition arc radius meets the requirements $(R_a, R_f > 0.4 \times m)$ [7].

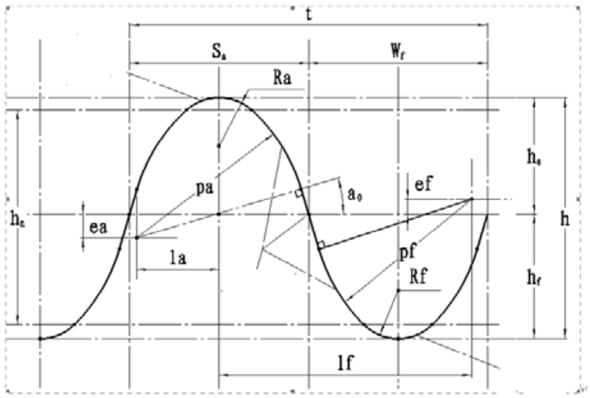

Fig.1 The basic rack tooth profile of the harmonic with double-circular-arc

a_0—tooth profile pressure angle, h—the full tooth height, h_a—addendum, h_f—dedendum, p_a—convex tooth profile arc radius, p_f—concave tooth profile arc radius, e_a—shift amount of the center of the convex tooth profile, e_f—shift amount of the center of the concave tooth profile, l_a— offset of the center of the convex tooth profile, l_f—offset of the center of the concave tooth profile, R_f-dedendum transition arc radius, R_a-addendum transition arc radius, S_a—convex tooth pitch line thickness ($S_a = \pi m / 2$), W_f—concave tooth pitch line cogging width ($W_f = S_a$), m—modulus, t—pitch ($t = \pi m$), h_n—tooth profile work section height.

The Selection Of The Main Tooth Profile Parameters. ***The Height Of Tooth Profile Work Section***. In the condition that the harmonic drive is in normal work, in order to improve its carrying capacity hn should be increased as big as possible. However, because the FS and the CS are likely to have the addendum interference, restricted by the processing condition, hn is usually $(1.4 \sim 1.6) \times m$.

Pressure Angle. By analyzing the involute tooth profile[8], it can be concluded that gear teeth shifting increases the average pressure angle, with which the meshing depth and the meshing area of the harmonic gear drive will increase. The involute tooth profile shifting is to adjust the average pressure angle to avoid tooth profile interference and improve the meshing quality. The circular arc gear can't make a big shift, so choosing a reasonable pressure angel a_0 is very important. Research shows that when the harmonic ratio is between 100-150, the optimal range of the basic rack tooth profile pressure angle is 15-17deg, when the harmonic ratio is between 50 and 100, its optimal range is 17-20deg.

The Arc Radius Of The Concave And Convex Tooth. Research shows that in the harmonic gear drive the abrasion loss of the gear tooth is the largest in the addendum part, and reduces from the addendum to the dedendum, and the FS has the largest abrasion loss. So, the radius of the FS addendum arc should be larger than the one of the dedendum, whose difference value Δp should not be bigger than 5%. The proper radius difference Δp can increase gear teeth contacting area, decrease tooth surface specific pressure, and improve the pitting resistance of the gear tooth and deduce the abrasion after the harmonic drive running-in. The convex tooth of the basic rack tooth profile conjugates to the FS dedendum arc, and the concave tooth conjugates to the FS addendum arc, so $p_a < p_f$, and the optimal range of p_a is $(1.4 \sim 1.6)\times m$, and the optimal range of p_f is $(1.00 \sim 1.05)\times p_a$.

Center Coordinates Of The Convex Tooth And Concave Tooth Circular Arc.The center coordinates of the convex tooth and concave tooth arc play a key role in the design of basic tooth profile, whose slight change will have a serious impact on the meshing quality and transmission performance of the harmonic drive. The optimal range of the center coordinate is ($l_a, l_f = (0.70 \sim 0.75)\times m$, $e_a, e_f = (0.1 \sim 0.3)\times m$).

Optimizing Of The Basic Rack Tooth Profile. First the basic rack tooth profile parameters were primarily chosen according to the selecting methods of the basic rack tooth profile's main parameters, and the FS tooth profile conjugate to the basic rack tooth profile (the work tooth profile) was solved with the tooth profile normal line method. Second the FS addendum thickness was checked to see whether it was no less than 0.2 times the modulus, if it wasn't, then returned to modify the basic rack tooth profile dedendum arc parameter ef. If it was, the CS tooth profile conjugate to the FS tooth profile was solved with the harmonic gear drive envelope theory; then the harmonic gear drive meshing area was verified to see whether it was greater than 30% and whether the tooth profile enveloped by the FS addendum arc contained the tooth profile enveloped by the FS dedendum arc, and the largest gap between the two tooth profiles is no bigger than $0.001\times \mathrm{m}$; if it wasn't, then returned to modify the basic rack profile addendum arc parameters la and ea; if it was, then the meshing of the tooth profile was checked.

Solving The Tooth Profile Of The CS and FS. ***Solving The FS Tooth Profile .***The FS tooth profile conjugate to the basic rack tooth profile was solved based on the parametric equation of the basic rack tooth profile (x_1, y_1). The included angle γ between the tangent line of any point on the basic rack tooth profile and x_1 coordinate axis was calculated. To make this point a contact point, the basic rack tooth profile had to be moved a distance $L = x_1 + y_1 + \tan\gamma$, then the FS gear was correspondingly turned away $\varphi_2 = L / r_2$ (r_2 was the radium of the FS reference circle) from the starting position , the FS tooth profile conjugate to the basic rack tooth profile was calculated with the coordinate change (Eq.1).

$$\begin{cases} x_2 = +x_1\cos(j_2) + y_1\sin(j_2) + r_2(\sin(j_2) - j_2\cos(j_2)) \\ y_2 = -x_1\sin(j_2) + y_1\cos(j_2) + r_2(\cos(j_2) + j_2\sin(j_2)) \end{cases} \tag{1}$$

Solving The Tooth Profile Of The CS. According to the present technological level, the circular ring shape deformed by the four force action was chosen as the original characteristic curve in the harmonic gear drive with double-circular-arc. With this curve, according to the ratio speed, the different optimal angle of β could be chosen to have the best meshing quality in the harmonic drive.

1) When $0 \le \varphi \le \beta$

$$\begin{cases} \omega = [\omega_0 / (A - 4/\pi)][A\cos\varphi + \varphi\sin\beta\sin\varphi - 4/\pi] \\ v = -[\omega_0 / (A - 4/\pi)][A\sin\varphi + \sin\beta(\sin\varphi - \varphi\cos\varphi) - (4/\pi)\varphi] \\ \theta = -\left\{\omega_0 / [r(A - 4/\pi)]\right\}[2\sin\beta\sin\varphi - (4/\pi)\varphi] \\ A = \sin\beta + (\pi/2 - \beta)\cos\beta \end{cases} \tag{2}$$

2) When $\beta \le \varphi \le \pi/2$

$$\begin{cases} \omega = [\omega_0/(A-4/\pi)][B\sin\varphi + (\pi/2-\varphi)\cos\beta\cos\varphi - 4/\pi] \\ v = -[\omega_0/(A-4/\pi)][-B\cos\varphi + (\pi/2-\varphi)\cos\beta\sin\varphi - \cos\beta(\cos\varphi+\varphi\sin\varphi) - (4/\pi)\varphi + 2] \\ \theta = -\{\omega_0/[r(A-4/\pi)]\}[2\cos\beta\cos\varphi + (4/\pi)\varphi - 2] \\ B = \cos\beta + \beta\sin\beta \end{cases} \tag{3}$$

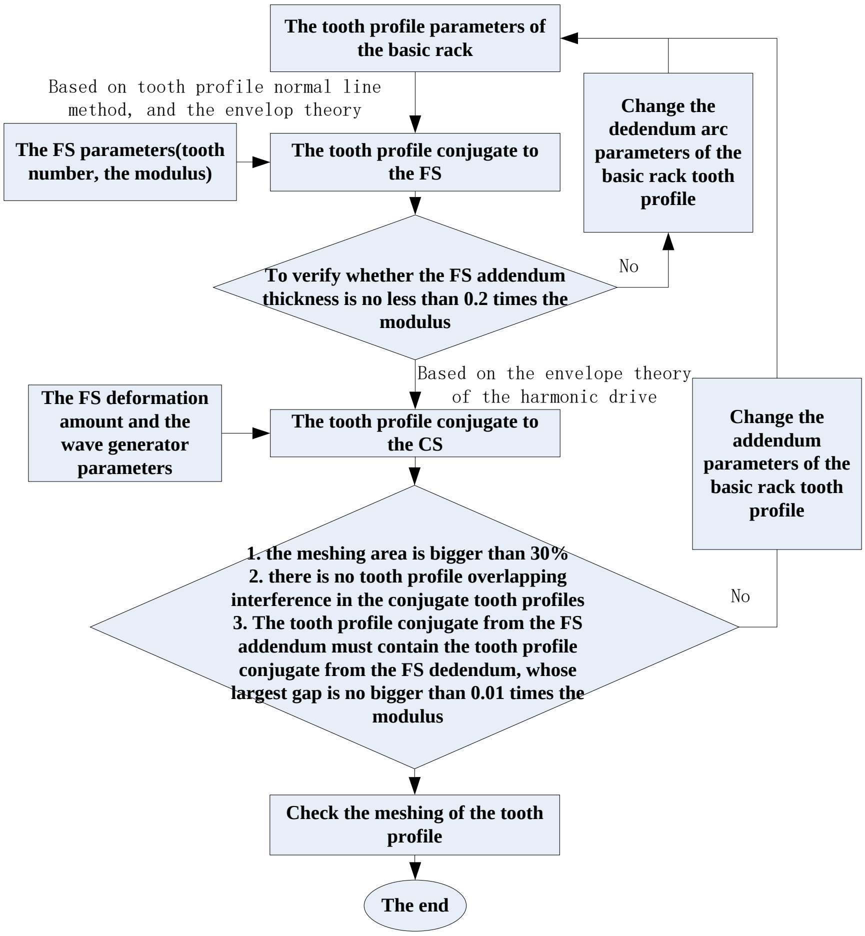

Fig.2 Optimization of the basic rack tooth profile

Suppose the fixed coordinate system $o-xy$ was fixed with the wave generator, the origin was the symmetry center of the wave generator, the moving coordinate system $o_g - x_g y_g$ was fixed with the CS, the origin coincided with the center of the wave generator, y_g axis coincided with the axis of symmetry of the CS cogging, as is shown in Fig 4, it was assumed that the fixed CS of the wave generator rotated the angle β_2, the FS rotated the angle β_1, the FS teeth and the CS teeth were conjugate. The undeformed end of the FS rotated the angle β, the polar radius of the original curve was ρ, the tooth symmetry axis of the FS rotated the angle μ, the angle between y_g and y_r was ϕ, the angle between y_g and the radius vector was γ.

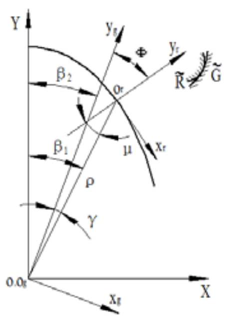

Fig.3 The schematic diagram of the coordinates' relationship

$$
\begin{cases}
\rho = r + \omega & \mu = (d\omega / d\beta) / r \\
\beta_1 = \varphi + v / R & \gamma = \beta_1 - \beta_2 \\
\beta_2 = (Z_R / Z_G)\varphi & \Phi = \gamma + \mu
\end{cases} \tag{4}
$$

Assuming that the wave generator was fixed, the FS was active and the CS was driven, when the longitudinal axes of the three coordinate systems were coincided, to get solution conveniently, the curve of the FS tooth profile was expressed in the parameters at first: ($x_r(t)$, $y_r(t)$). The CS tooth profile conjugate to the FS tooth profile was obtained according to the conjugate tooth profile and the envelope theory of the harmonic drive.

$$
\begin{cases}
x_g(t,\varphi) = +x_r \cos\Phi + y_r \sin\Phi + \rho \sin\gamma \\
y_g(t,\varphi) = -x_r \sin\Phi + y_r \cos\Phi + \rho \cos\gamma \\
\dfrac{\partial x_g}{\partial t} \cdot \dfrac{\partial y_g}{\partial \varphi} - \dfrac{\partial x_g}{\partial \varphi} \cdot \dfrac{\partial y_g}{\partial t} = 0 \\
\Phi = \gamma + \mu
\end{cases} \tag{5}
$$

The Calculation Example. Took the harmonic gear drive with 90 type double-circular-arc for example, referring to the selection methods of the basic rack tooth profile's main parameter, the basic rack tooth profile parameters were selected as followed: $a_0 = 16.5^\circ$, $h = 0.76\text{mm}$, h_a=0.3675mm, h_f=0.3925mm, p_a=0.5426mm, p_f=0.5555mm, e_a=0.0747mm, e_f=0.0470mm, l_a=0.2610mm, l_f=0.8482mm, R_f=R_a=0.1530mm, m=0.36mm, h_n=0.576mm, $W_f = S_a = \pi m / 2$, $t = \pi m$. The basic parameters of the double-circular-arc harmonic gear drive were: the FS tooth number $Z_R = 260$, the CS tooth number $Z_G = 262$, the modulus m=0.36mm, the maximum deformation $\omega_0 = 0.36\text{mm}$, the radius of the FS undeformed characteristic curve $r = 45\text{mm}$, the original characteristic curve was the circular ring shape deformed by the four force, $\beta = 25\deg$ (The FS stress would change with the angle β, when $\beta \approx 25^\circ$, the FS stress was the least.)

The Determination Of The FS Tooth Profile. The FS tooth profiles respectively conjugate to the addendum, tangent and dedendum of the basic rack tooth profile were obtained with the tooth profile normal line method and coordinate transformation. As is shown in Fig.3 the FS tooth profile were composed of three tooth profiles respectively conjugate to three parts of the basic rack tooth profile.

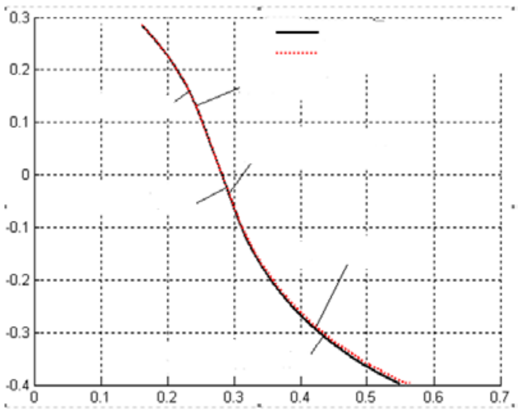

Fig. 4 The meshing curve of the basic rack tooth profile and the FS tooth profile

By calculation, the FS addendum tooth thickness was 0.3205mm, far greater than 0.2 times the modulus, which met the design requirements.

The Determination Of The CS Tooth Profile. The parameters t and φ were in the implicit function which were difficult to express in elementary function, so the CS tooth profile could only be obtained with numerical method. By numerical calculation, it was found that the envelope curve formed by the FS motion was just the CS tooth profile. Fig.5 illustrated that the FS had two envelope curves, the inside and the outside, and the CS tooth profile was the outside envelope curve. Fig.6 illustrated that the CS addendum tooth profile enveloped by the FS addendum tooth profile contained the CS addendum tooth profile enveloped by the FS dedendum tooth profile, and the biggest gap between the two CS addendum tooth profiles enveloped was 0.0027mm, so that the harmonic drive of this basic rack tooth profile had a relatively even meshing backlash, a quite long tooth profile contacting area and more meshing tooth numbers at the same time. In conclusion, the tooth profile of the CS addendum and deddendum was formed by the FS addendum tooth profile's enveloping, the middle tooth profile of the CS was formed by the FS middle involute tooth profile's enveloping.

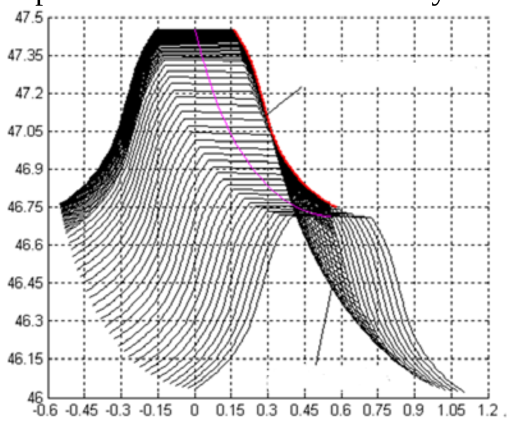

Fig.5 The CS curve enveloped by the FS

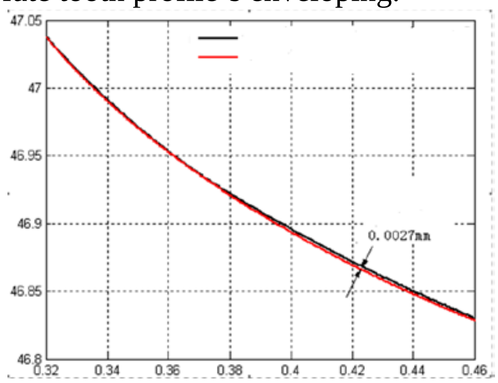

Fig. 6 The comparison diagram of the CS addendum tooth profiles enveloped by the FS addendum and deddendum respectively

The 1/4 FS tooth meshing the CS tooth was simulated in the MATLAB, as is shown in Fig.7. There was no tooth profile overlap interference when the CS and the FS were in meshing drive. As is shown in Fig.8 when the polar angle φ=0deg, the CS addendum tooth profile and the FS dedendum tooth profile were basically in coincidence. When the polar angle φ ≈62deg, meshing started, the theoretical meshing area was 69% in the double-circular-arc harmonic drive. In the whole meshing process, the teeth were always conjugate, without edge contacting. Compared with the finite conjugate motion of the involute tooth profile harmonic gear drive, this basic rack tooth profile harmonic gear drive could fully realize the whole tooth conjugate contact.

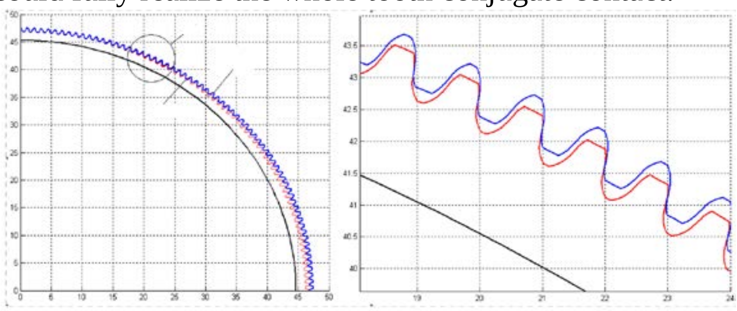

Fig.7 The meshing diagram of the 1/4 FS tooth and the CS tooth

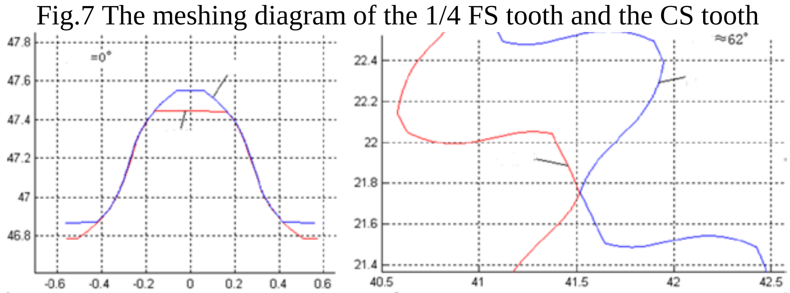

Fig. 8 The relative position graph of the CS and FS tooth when the polar angle $\varphi = 0^{\circ}$ and $\varphi \approx 62^{\circ}$

Conclusion

The normal tooth profile of the hob basic rack is creatively chosen as the basic rack tooth profile, the CS and FS are produced by the most mature gear shaping and hobbing processing technology. So that the anufacturability of the harmonic gear drive with the double-circular-arc are improved greatly. Based on the selection methods of the basic rack tooth profile main parameters, the design method and process of the basic rack tooth profile is summarized. It is checked that the FS tooth profiles of the harmonic gear drive with the basic rack tooth profile are conjugated respectively by the work tooth profiles of the basic rack tooth profile; the CS addendum and deddendum tooth profiles are formed by the enveloping of the FS addendum and deddendum, the CS middle tooth profile are formed by the enveloping of the FS middle involute tooth profile. It is concluded by the simulation that the theoretical meshing area of the basic rack harmonic gear drive can reach 69%, greater than that of the international top products, and it can truly realize the whole tooth conjugate, which is the most effective way to improve the comprehensive performance of the harmonic gear drive products.

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